1. The table below gives the Grundy values for all pairs of pile sizes from up to 10 per pile for Whytoff's game. For example, the Grundy value of the position $(3, 10)$ is 8. Fill in the unfilled squares in order to determine the Grundy value for the initial position (11,11). Recall that in Whytoff's game, at each turn a player can either take any number of counters from **one** pile or the **same** number of counters from two piles.

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	
1	1	2	0	4	5	3	7	8	6	10	11	
2	2	0	1	5	3	4	8	6	7	11	9	
3	3	4	5	6	2	0	1	9	10	12	8	
4	4	5	3	2	7	6	9	0	1	8	13	
5	5	3	4	0	6	8	10	1	2	7	12	
6	6	7	8	1	9	10	3	4	5	13	0	
7	7	8	6	9	0	1	4	5	3	14	15	
8	8	6	7	10	1	2	5	3	4	15	16	
9	9	10	11	12	8	7	13	14	15	16	17	
10	10	11	9	8	13	12	0	15	16	17	14	
11	11	9	10	7	12	14	2	13	17	6	18	15
12												

2. Now consider the composite game $G_1 \oplus G_2 \oplus G_3 \oplus W(12, 11) \oplus N(3, 5, 7, 9)$ where $G_1 = N(20; 1, 3, 5), G_2 = N(20; 1, 2, 5)$ and $G_3 = N(20; 1, 2, 6)$ are the games defined in assignment 5, and $W(12, 11)$ is the game in problem 1 above. Of course, the game $N(3, 5, 7, 9)$ is itself a composite of the four one pile nim games $N(3), N(5), N(7)$ and $N(9)$. That is, $N(3, 5, 7, 9) = N(3) \oplus N(5) \oplus N(7) \oplus N(9)$. This composite game is played as follows: at each turn a player selects one of the five component games and make a legal move in that game. For example, denoting the initial position by $(20, 20, 20, (12, 11), 3, 5, 7, 9)$, the first player could move to $(20, 20, 20, (11, 10), 3, 5, 7, 9)$, since that corresponds to taking one counter from each of the two Whytoff piles. Compute the Grundy value of the composite game. If it is positive, find a winning move. Find a winning rejoinder to the move that results in $(20, 20, 20, (11, 10), 3, 5, 7, 9)$. Are there other winning moves?