1. For each of the Bouton's Nim games below, find the binary representations of the pile sizes and use this to determine the Grundy value of the game. If the game is not in $\mathcal{S}$, find the move that results in a safe position. Tell when there is more than one winning play.

   (a) $N(17, 19, 22, 25, 29)$

   (b) $N(117, 119, 212, 215, 219)$

   (c) $N(1, 3, 9, 27, 81)$

   (d) $N(1, 2, 4, 8, 16, 32, 64)$

2. In Laskar's Nim, we modify the rules of Bouton's Nim to include another type of move– a splitting of a pile. Instead of taking counters from a pile, a player may choose to split the pile into two smaller nonempty piles. Analyze the games $LN(3)$, $LN(3, 4)$, $LN(2, 3, 4)$, and $LN(1, 3, 5, 7)$. Hint: start with the simplest games and work your way up.