

Cram

by Meghan Corcoran

Combinatorial Games

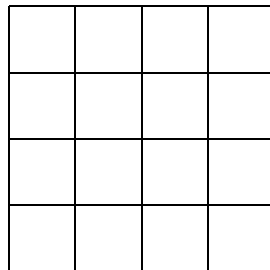
Instructor: Dr. Harold Reiter

Introduction. Cram is a game with two players and a square or rectangle board. The board can be of any length or width the players want it to be. Using a one by two, or dominoes, game pieces, the players take turn putting a domino down on the board. The domino must cover exactly two squares and they can not overlap one another. The player that makes the last possible move is the winner. The alternative ways of playing cram is to use triominoes instead of dominoes or the player to make the last move is the loser not the winner.

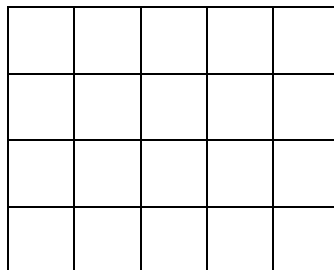
The subject that I have chosen to write my report on is Cram. Cram is a type of Combinatorial Game, which means it is a two-person game. It also means that the players take turns making moves and the player that makes the last move is the winner. There are two kinds of Combinatorial Games, partisan and impartial. Cram is an impartial game. An impartial game is where a player can make any move on the board.

Cram is played on a square or rectangular board of any length or width chosen by the players. Players use one by two pieces called dominoes to make their moves. The dominoes can not overlap and must take up only two squares. To win, as in all impartial games, the player that puts down the last domino considered the winner. Some other games that have the same basic rules are is Plugg, Dots-&-Pairs, and Impartial Domineering.

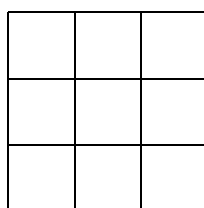
There are three different kinds of boards that Cram can be played on, an even number side by an another even number side,



an odd number by an even number,

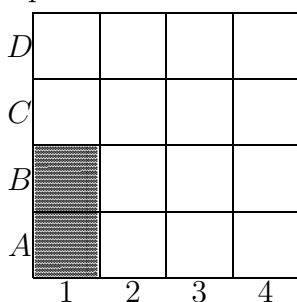


and an odd number by an odd number.

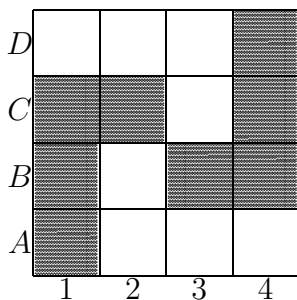


Each board brings about a different way of winning each time one plays.

The first board that will be discussing is the even number by even number. To win every time the player must go second, but first he must find the point in the very middle of the board. The first player puts down the first domino in whatever position they choose. Where ever the 1ST player moves the second player will rotate 180 degrees around the point and put there domino down. For example, if player number 1, Bob, puts his domino down in squares A1 and B1, player number 2, Jim, would put his domino down in squares D4 and C4.

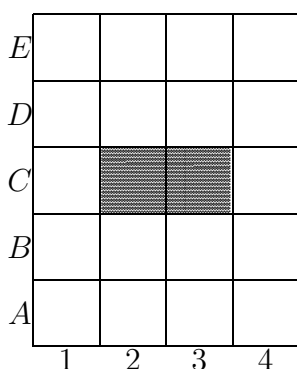


If Bob put his domino into squares C1 and C2, Jim would place his in B3 and B4.

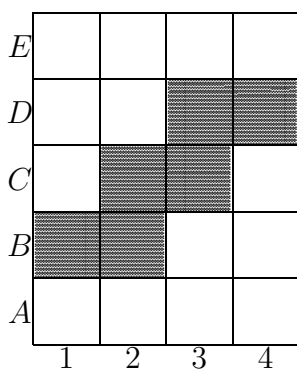


This method of play is continued until player number two, Jim, wins the game.

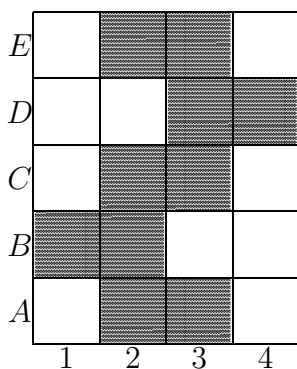
On an even by odd board, like a four by five, the key to winning the game is to make the first move. The move has to be in the two center squares, C2 and C3.



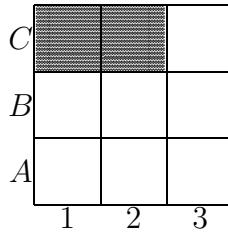
After those squares are filled the game has the same strategy as the even by even. Every move that the second player makes, the first player rotates the board 180 degrees around the two center squares. After Jim places his domino into C2 and C3, Bob takes his turn. If Bob put his domino in B1 and B2, Jim would place his into D3 and D4.



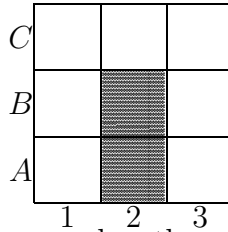
Then, if Bob put his domino in A2 and A3, then Jim would place his in E2 and E3.



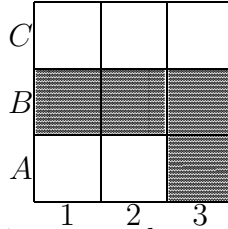
This play would go on until Jim wins the game. The final type of board that Cram can be played on is an odd by odd board, or a three by three. On a three by three board, the winner has to go second. There are only two possible moves for the first player to put their domino. Those moves are the two side squares,



or a middle and side square.



Whatever move the first person makes the second player would do the other move. If Bob placed his domino in B1 and B2, Jim would place his into either B3 and C3 or A3 and B3.



This causes the board to split into a one by two board and a one by three board. Both of the boards contain just one move, always letting Jim or the second player win.

The reason why these strategies work is because every combinatorial game has a Grundy Value. Grundy Value is defined as the smallest nonnegative integer in a set of numbers. For example, in the set 0,1,5,9 the Grundy Value is 2. Grundy Value is needed in combinatorial games to find what moves are safe and which ones are not. All the moves with a Grundy Value of 0 is considered safe and all other are unsafe. The only move from a safe position is into a unsafe position but from a unsafe position you can move into a safe or unsafe position.

In the four by four board, the starting game has a Grundy Value of zero. To win one would want to go second because the only move for the first player to make a move is into an unsafe position. Then the second player can make their move into a safe position. This pattern then goes on until the second player wins. In the three by three game, the grundy value is also zero. The rules for how to win on the four by four apply to the three by three.

In the four by five game the Grundy Value is a little different. The Grundy Value for the blank board is one not zero. In this game one would want to go first to make the move into a safe position, or with a Grundy Value of zero. The second player would then make a move into an unsafe spot.

After one has become an expert in the game of Cram, they can alter the rules to make it harder. They can do this by using triominoes, which are one by three pieces, instead of dominoes. Another change would for this game would be to make the player who makes the last move the loser instead of the winner. A third possible change would be to play on a non-square or rectangular board. The shape could be a staircase or a pyramid.

Cram is a complex game of skill. It takes great amounts of time and dedication to understand all versions of this one game. Over trial and error, This game can be successfully understood by anyone. Complete understanding can bring about great joy to those who can win every time.