Moneybags

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Combinatorial Games

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Section 1. Introuction. The world of math explores many realms of analytical diversity. One of the most distinguished analytical forms of data comes from Combinatorial Games and its theories. Combinatorial games are two player games which much satisfy the following conditions: players alternate extracting counters from a finite collection according to some set of rules and the last players to remove of place a counter wins. The game, which shows much interest, in that of the game strategy and the winning prize, is Moneybags. Moneybags, also know as The Silver Dollar Game is credited to N. G. de Bruijn. Moneybags is a combinatorial game made up of a strip of paper (any length) marked into squares. The game is played by laying out the strip of paper and placing any number of markers (preferably money coins) anywhere on the square. There may be one marker occupying a square and any number of squares empty. Each player takes turns moving one marker to the left but not passing another marker. The leftmost square is the moneybag square. The moneybag counts as a square and the person who places the last coin in the moneybag wins and gets to keep all the money! In almost all combinatorial games, one, if winning is the goal, must obtain a strategy. Usually the strategies come from an understanding of the games' Grundy Values and the Minimum Excludant [MEX]. In Moneybags, finding the Grundy Values and the MEX are extremely helpful. With the use of the above-mentioned strategies, one will most likely conclude that the second player will win. Moneybags is more than just a game in which the players just take turns moving money closer toward the moneybag square itself. It is also more than just a game of chance. Moneybags is a game of concentration, a game in which one may master, if one knows how to master it. In order to find the winning strategic movements in Moneybags, one must set up a table. In this table, one will include Grundy Values. Grundy values are simply numbers assigned to each possible position. One would set up a table starting from 0 and work their way up to the highest number of counters. To further carry out the process of achieving Grundy Values, one would also need to find the mex (Minimum Excludant) The minimum excludant is basically the smallest number from a pattern of numbers. For example:

In the following list of numbers...0,1,3,4,5,6,7,8,9,10 The mex (minimum excludant) would be 2. The mex would be two because out of the above arrangement of numbers two is the minimum number excluded from the list.

Other examples are:

$$Mex(0, 1, 2, 4) = 3$$

$$Mex(1, 2, 3, 4, 5) = 0 \text{ and}$$

$$Mex(4, 2, 3, 0) = 1$$

Grundy Values are computed from the mex of a given array of numbers. By taking the previous Grundy Values from previous positions that may affect the position in which one is looking for a Grundy Value, then one may continue for

their search on values.

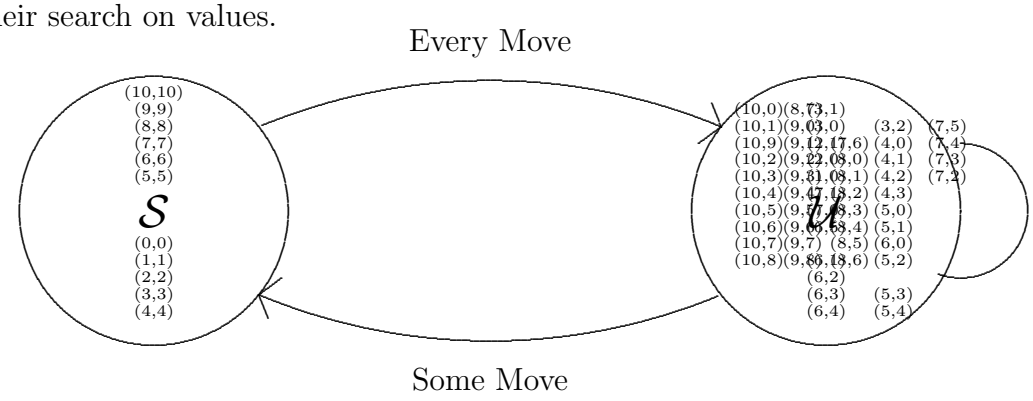

Fig. 2

The Grundy Values for (2,0) is 2 and the Grundy Value for (2,2) is 0. Thus the Grundy Value (2,1) will be the Grundy Value of (2,0) and (2,2). Thus 0,2 MEX=1

The Grundy values then tell you all the safe and unsafe positions. All the safe positions are those that have a 0 Grundy Value and all the unsafe positions are those with positive values other than 0. It is helpful to chart the safe and unsafe positions in some kind of chart. The most reasonable way to chart the results would be that of a flow chart. The player wins if a player stays in a safe position that way one's opponent is left on a losing streak.

	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1		0	1	2	3	4	5	6	7	8	9
2			0	1	2	3	4	5	6	7	8
3				0	1	2	3	4	5	6	7
4					0	1	2	3	4	5	6
5						0	1	2	3	4	5
6							0	1	2	3	4
7								0	1	2	3
8									0	1	2
9										0	1
10											0

Steps in finding Grundy Values:
1. Calculate the total number of squares in a game of Moneybags, starting with zero.
2. Create a table, labeling the x-axis columns with the number of total squares.
3. Calculate the number of counters placed on the game board. Label those on the y-axis.
4. Starting with the first position (0,0) see if you can move to any other position. In this case you cannot. Then work your way up to (1,0) (1,1) (2,0)..... It is not necessary to calculate the Grundy Value of a position if it falls after a number with the same coordinates.
EX: From (1,1) a player can move to (1,0) or (0,0). We know that (0,0) has a value of 0 because there are no more movements that cannot be made but (1,0) has a value of 1 because the next player can move to the (0,0) position. 5. Continue to calculate the Grundy Values of all the positions. One would use mex to find the other values in a column or row. Note: Sometimes instead of using a table, one may prefer to actually draw the game board several times and show all the position changes of each move, thus calculating the Grundy Value. When doing so an individual can see all the positions in which they can move to be safe and win a game although it is a more tedious way in finding the pattern. (Table MB-4)

After calculating the all the values to play Moneybags, it is much easier to win the game. The player with the strategic plan of Grundy Values will win according to the Grundy Value chart. It is really not definite if the first or second player will win. However, who knows Grundy Values and knows how to stay in safe positions will win the game and also be rich!