

1. Recall the subtraction game in which two players start with two positive integers  $a$  and  $b$  written on a board. The first player subtracts one of the numbers on the board from a larger one, and write down the new difference. At each stage, the next player finds a positive difference between two numbers that is not already written on the board and writes it on the board. The first player who cannot find a new positive difference loses. For each of the pairs listed below, write down all the numbers that will eventually appear on the board, and use this information to state whether the game will be won by the first player or the second.
  - (a) 35 and 42
  - (b) 36 and 42
  - (c) 39 and 42
  - (d) 40 and 42
2. One problem we discussed in class is the problem of determining the sum of all the numbers that appear in the table of digital products. We saw that we could use a geometrical approach that treats the numbers to be added as areas of rectangles which can be used to build a larger rectangle. Use this idea to prove that for any positive numbers  $x$  and  $y$ ,

$$(x + 1)(y + 2) = xy + y + 2x + 2.$$

3. Two children take turns breaking up a 6 square by 8 square rectangular chocolate bar. They break the bar only at the divisions between the squares. If the bar breaks into several pieces, they keep breaking one piece at a time until only the squares remain. The first player who cannot make a break is the loser. Who will win?
4. There are three piles of stones, one with 10 stones, one with 15 stones and one with 20 stones. At each stage, a player can choose a pile and divide it into two smaller piles. The loser is the first player who cannot do this. Who wins and why?
5. Two players take turns putting castles (rooks) on a chess board so they cannot capture each other. The winner is the last to put down a castle. Who will win and how? Castles move along rows and along columns.
6. Ten 1's and ten 2's are written on a board. In one turn, a player may erase any two digits. If they are identical, they are replaced by the single digit 2 and if they are different, they are replaced by the single digit 1. The first player wins if a 1 is left at the end and the second wins if a 2 is left. Who wins and why?