Pigeon Hole Principle Problems   Math 1102

1. In a standard deck of 52 cards, what is the smallest number of cards you must draw to guarantee that you will have at least one pair?

2. In a standard deck of 52 cards, what is the smallest number of cards you must draw to guarantee that you will have 5 cards of one suit?

3. Is it true that from any 30 different natural numbers, not greater than 50 one can choose a pair such that one number in the pair is twice the other.

4. Prove that among any set of 51 positive integers less than 100, there is a pair whose sum is 100.

5. Five points are selected from the square defined by $\{(x, y)| -1 \leq x \leq 1, -1 \leq y \leq 1\}$. Prove that two of these are within $\sqrt{2}$ of each other.

6. 289 points are scattered inside a square with a side of one foot. Prove that some subset of three of these points can be covered by a square with side one inch.

7. The integers from 1 to 10 are randomly distributed around a circle. Prove that there must be three neighbors whose sum is at least 17. What about 18? What about 19?

8. The digits $1, 2, 3, \cdots, 9$ are divided up into three groups. Prove that the product of the numbers in one of the groups must exceed 71.