Prime Numbers Math 1102

1. Find the prime factorization of each of the following numbers:
   (a) 1001
   (b) $6!+1$. Recall that the notation $n!$ means multiply together all the positive integers up to and including n. Thus, for example $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$.
   (c) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 1$

2. Let p stand for a prime that is larger than 3-that is, $p > 3$. Can $p + 1$ ever be prime? Why or why not?

3. Are there infinitely many nonprimes? Explain.

4. Let $M = (1 \cdot 2 \cdot 3 \cdot \cdots \cdot 1000) + 1$
   (a) Is M necessarily prime?
   (b) What can you say about the prime factors of M? Why?
   (c) Let $S = (2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot \cdots \cdot 991 \cdot 997) + 1$-that is, S is one more than the product of all the primes less than 1000. Is S necessarily prime?
   (d) What can you say about the prime factors of S? Why?

5. Prove that there are infinitely many primes.

6. Recall from class that for any three digit number $\overline{abc}$, the result of multiplying by first 7 then 11, then 13 is $\overline{abc, abc}$. Compute the product $7 \cdot 11 \cdot 13$. Use your answer to explain the strange result above.