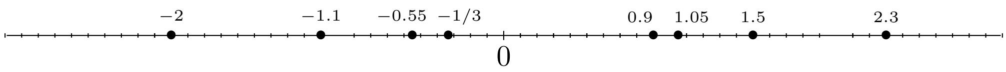
Rational Numbers Problems — Math 1102



1.

2. $10 \times 3.14 = 31.4$, $1000 \times 0.123123\ldots = 123.123\ldots$, $10 \times 0.49999\ldots = 4.9999\ldots$, $\frac{98.6}{100} = \frac{986}{1000}$ and $\frac{0.333...}{10} = 0.0333\ldots$.

3. $\frac{1}{9} = 0.111\ldots = 0.\overline{1}$.

4. Let $M = 0.4999\ldots$. Then $10M = 4.999\ldots$ and $9M = 10M - M = 4.999\ldots - 0.499\ldots = 4.5$, so $M = \frac{4.5}{9} = \frac{1}{2} = 0.5$.

5. Being irrational means the number cannot be expressed as the quotient of two integers.

10. One example is $5.701001000100001\ldots$ where the block of 0s gets longer by 1 each time.

22. Let $x = 5.63121212\ldots$. Then $100x = 563.121212\ldots$ and $99x = 100x - x = 563.1212\ldots - 5.631212\ldots = 557.49$. Therefore $x = \frac{557.49}{99} = \frac{55749}{9900} = \frac{18583}{3300}$, when reduced to lowest terms.

23. Let $x = 0.010101\ldots$. Then $100x = 1.01010101\ldots$ and $99x = 1$. Therefore $x = \frac{1}{99}$.

24. Let $x = 71.23999\ldots$. Then $10x = 712.3999\ldots$ and $9x = 641.16$. Therefore $x = \frac{641.16}{9} = \frac{64116}{900} = \frac{7124}{100} = 71.24$.

37. Let $x < y$ be two real numbers. Let $d = y - x$. Of course, $d > 0$. If d is irrational, let d' be a rational number less than d obtained by truncating the digits after the second nonzero digit of d. For example, if $d = 0.000123....$ then $d' = 0.00012$. Now if x is rational, then $x + d$ is irrational and between x and y and $x + d'$ is rational and between x and y. On the other hand suppose x is irrational. Then $x + d'$ is an irrational between x and y. We can find a rational number between x and y by taking the decimal representation of $x + d'$, and going out past the position of the third nonzero digit of d', at which point we truncate (make all the digits zero), the number $x + d'$.