Mathematical Thinking **Test 1**

October 12, 2006 Your name ______________________

Throughout this test you must **show your work**. Answers without supporting work are generally worth about one fourth credit. The total number of points available on this test is 143.

1. (8 points) Fill in the grid so that every row, every column, and every 2×2 box contains the digits 1 through 4.

2			4
		3	
	4		
1			

2. (8 points) Decimals and Fractions

   (a) What is 75% of 30?

   (b) What is 30% of 75?

   (c) The number 75 is 30% of what number?

   (d) Find two numbers between $7 + \frac{2}{5}$ and $7 + \frac{41}{100}$.

3. (12 points) Costco Green Peas
   Recall that Costco Green peas come in cans which are packaged 6 cans to a package, 6 packages to each box, 6 boxes to each crate, 6 crates to each order, and 6 orders to each truckload. For each of the number orders given below, find a way to put the order together without using more than 5 of any unit or measure. In other words, find the base 6 representation of each of the numbers.
   (a) 2006
   (b) 30000
   (c) 9331

4. (8 points) Construct the base 6 addition and multiplication tables for base 6 digits.

+	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

×	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

5. (8 points) Use the base 6 addition and multiplication tables from the previous problem to perform the arithmetic shown.
   (a) $1302_6 - 323_6$
   (b) $1302_6 \times 323_6$

6. (10 points) Explain in detail why the number of primes must be infinite.

7. (12 points) Notice that $92 + 29 = 121$ and $92 - 29 = 63$. The first one is a multiple of 11 and the second a multiple of 7 and of 9.

   (a) Is it true that for any two digits, a and b, $\underline{ab} + \underline{ba}$ is a multiple of 11? Explain why or why not.

   (b) Is it true that for any two digits, a and b, $\underline{ab} - \underline{ba}$ is a multiple of 9? Explain why or why not.

   (c) Is it true that for any two digits, a and b, $\underline{ab} - \underline{ba}$ is a multiple of 7? Explain why or why not.

8. (10 points) The first Fibonacci number that is a multiple of 9 is $F_{12} = 144$ [note that $144 \equiv 1 + 4 + 4 \equiv 0 \pmod 9$]. What are the subscripts of the next two Fibonacci numbers that are multiples of 9. There is no need to find the value of the number, just the subscripts. The subscript of the twelfth Fibonacci number F_{12} is 12.

9. (8 points) Recall the subtraction game in which two players start with two positive integers a and b written on a board. The first player subtracts one of the numbers on the board from a larger one, and writes down the new difference. At each stage, the next player finds a positive difference between two numbers on the board that is not already written on the board and writes it on the board. The first player who cannot find a new positive difference loses. For each of the pairs listed below, decide how many numbers will be on the board at the end of the game. Use this information to state whether the game will be won by the first player or the second.

   (a) 135 and 141

   (b) 195 and 205

10. (8 points) **Finding the unknown digit.**
    Let $N = \underline{abcde}$ denote the five digit number with digits a, b, c, d, e and $a \neq 0$. Let $N' = \underline{edcba}$ denote the reverse of N. Suppose that $N > N'$ and that $N - N' = 5x014$ where x is a digit. What is x?

11. (15 points) A standard deck of 52 cards has four *suits*, clubs, diamonds, hearts and spades. Each suit has 13 *values* including ace, two, three, ..., ten, and three face cards, Jack, Queen and King. For each event listed below, find the number of cards that must be selected from a deck to guarantee that the event must happen.

    (a) The set contains a pair; that is, two cards of the same value.

    (b) The set contains three-of-a-kind; that is, three cards of the same value.

    (c) The set contains two aces.

    (d) The set contains two pairs (or four of a kind, which is, in a sense, two pair).

    (e) The set contains five spades.

12. (9 points) Find the prime factorization of each of the following numbers. Note that if a number is prime, it is its own prime factorization.

    (a) 481,481

    (b) 219

    (c) 177

13. (12 points) Use the Euclidean algorithm to solve the decanting problem for decanters of sizes 219 and 177. In other words, find integers x and y such that $gcd(219, 177) = 219x + 177y$.

14. (15 points) When the leftmost digit of a six digit number N is moved to the right end of the number, the new number N' is three times as big. Symbolically, $N = \underline{abcdef}$, $N' = \underline{bcdefa}$, and $N' = 3N$. Find a possible value of N.