During the last few years approximately 20% to 25% of my Math 1120 students have been unsuccessful in the course on account of their difficulty with algebra. Many find themselves able to understand the ideas of calculus, but unable to communicate their understanding because they mess up the algebra. The problems below represent topics with which a student beginning Math 1120 Business Calculus should be familiar. The problems should be completed in an hour **without** any aids, including a calculator. A score of 10 or more out of 15 means you're probably ready for this course.

1. Compute the value of $\frac{1}{2} - \frac{1}{3} + \frac{1}{7}$, and express your answer as a simple fraction.

   Solution: Rewrite the fractions so that each has a denominator of 42. $\frac{1}{2} - \frac{1}{3} + \frac{1}{7} = \frac{21}{42} - \frac{14}{42} + \frac{6}{42} = \frac{21-14+6}{42} = \frac{13}{42}$.

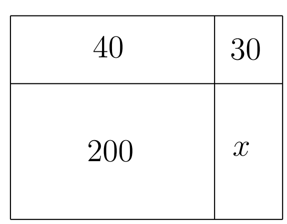
2. Find the y-intercept of the line that passes through the point $(2, 3)$ and has a slope of 6.

   Solution: Using point-slope form, the line is $y - 3 = 6(x - 2)$, which can be written in slope-intercept form as $y = 6x - 9$, so the y-intercept is -9.

3. One solution to $(x - 1)(x + 4) + (x - 1)(x + 7) = 0$ is $x = 1$. Find another solution.

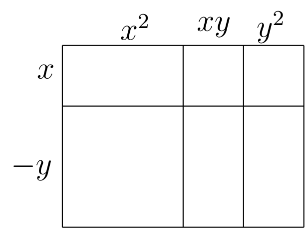
   Solution: We can factor to rewrite the equation as $(x - 1)(2x + 11) = 0$ so the other root is $-11/2$.

4. A rectangle is partitioned into 4 smaller rectangles. Three of the areas are shown. What is the area of the fourth one?

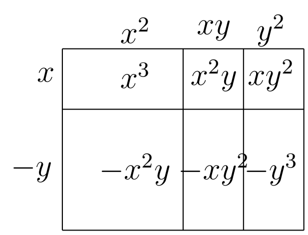


   Solution: The bottom left rectangle has five times the area of the top left one, so, by similarity, the bottom right rectangle has area five times the top right one. So $x = 150$. One way to albel the sides of the rectangle is 10 and 50 on the left and 4 and 3 on the top.

5. A rectangle with sides of length $x^2 + xy + y^2$ and $x - y$ is partitioned into 6 smaller rectangles. Find the area of the large rectangle in simplest terms.



Solution:



Now, note that four of the pieces can be disregarded and we get just the familiar factorization for the difference of two cubes, $x^3-y^3 = (x-y)(x^2+xy+y^2)$.

6. Rewrite the number $6^9 \cdot 9^6 \cdot 12^3 \cdot 3^{12}$ in the form $u^v \cdot w^z$.

   Solution: Rewrite each term as a product of powers of 2 and 3 to get $6^9 \cdot 9^6 \cdot 12^3 \cdot 3^{12} 2^9 \cdot 3^9 \cdot 3^{12} \cdot 2^6 \cdot 3^3 \cdot 3^{12} = 2^{15} \cdot 3^{36}$.

7. Put the numbers $2^{150}, 3^{100}, 5^{75}, 7^{50}$ in order from smallest to largest.

   Solution: Rewrite all four with an exponent of 25. $2^{150} = 2^{6\cdot 25} = (2^6)^{25} = 64^{25}$. Similarly, $3^{100} = 81^{25}; 5^{75} = 125^{25}$, and $7^{50} = 49^{25}$, so we can write $49^{25} < 64^{25} < 81^{25} < 125^{25}$.

8. A store reduces the price of an item by 20% and then gives a coupon for another 30% discount. What is the overall discount?

   Solution: If the original price is p, then the final price is $.70 \cdot .80p = .56p$, so the total discount is 44%.

9. A family goes on a vacation in their car. They average 60 miles per hour getting to their destination, and 40 miles per hour getting home. What is their overall average speed?

   Solution: For convenience, let's suppose that the distance they travel is 120 miles. This does not change the answer. Then they take 2 hours to get there and 3 hours to get back. The total distance traveled is 240 miles so the average speed is $240/5 = 48$ miles per hour.

10. Find a fraction (with whole number parts) between 1/3 and 2/5. Explain why your answer is correct.

    Solution: There are several good approaches to this. One would be to find the midpoint of the segment with ends 1/3 and 2/5. That is, find the average value, $(1/3 + 2/5)/2 = (10/30 + 12/30)/2 = 11/30$. Another way is to use the water bottle splitting idea. Three cyclers have one large bottle of water for a trip. Another group of five cyclers have two bottles to share. If all 8 agree to share equally, they each get 3/8 of a bottle. Yet another way to work this is to note that $1/3 < 0.34$ and $0.34 < 0.40 = 2/5$, so $34/100 = 17/50$ also works.

11. Let $f(x) = 2x+3$ and $g(x) = 2/(x+1)$. Build the composite function $f \circ g(x)$.

    Solution: Note that $f \circ g(x) = 2g(x) + 3 = 2(2/x + 1) + 3 = 4/(x + 1) + 3$.

12. What is the (implied) domain of the function $f(x) = \frac{3x+5}{x^2-4}$? Write your answer in interval notation.

    Solution: The only values we have to eliminate are the ones that cause division by zero, $x = \pm 2$, so the answer we want, in interval notation is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

13. Express $\frac{x^2+2x-3}{x^2-3x+2}$ in simplest form and draw its graph.

    Solution: We can factor both numerator and denominator to get $\frac{(x-1)(x+3)}{(x-1)(x-2)}$. Reducing this to $\frac{x+3}{x-2}$, you see that we get what is called a *hyperbola*. We'll learn more about these in the course.

14. What is the value of $\log_{12} 3^5 \cdot 2^{10}$?

    Solution: We can write $3^5 \cdot 2^{10} = (3 \cdot 2^2)^5 = 12^5$, so $\log_{12} 3^5 \cdot 2^{10} = \log_{12} 12^5 = 5$

15. Find all values of x for which $|x-1|+|x-2|+|x+7|=18$.

    Solution: Let $g(x)=|x-1|+|x-2|+|x+7|$. Checking the values of g at $x=1,2$ and -7, we get $g(1)=9, g(2)=8, g(-7)=17$. None of these work. If $x>2$, then $x-1, x-2$ and $x+7$ are all positive numbers so our equation becomes $x-1+x-2+x+7=3x+4=18$ and it follows that $x=14/3$. On the other hand, if $x<-7$, then all three of the quantities are negative and we have $1-x+2-x-7-x=-3x-4=18$, in which case $x=-22/3$.