May 6, 1998 **Name** ____________________

In the first eight problems, each part counts 8 points (total $12 \cdot 8 = 96$ points) and the other problems count as marked.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. A function f satisfies $f'(x) = 3x^2 - 2x$ and $f(1) = 2$. What is $f'(0)$?

   (A) $\boxed{0}$ **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 6

2. A function f satisfies $f'(x) = 3x^2 - 2x$ and $f(1) = 2$. What is $f(3)$?

   (A) 13 **(B)** 16 **(C)** 18 **(D)** $\boxed{20}$ **(E)** 24

3. A function G satisfies $G'(x) = 2x\sqrt{3x^2+1}$ and $G(0) = 11/9$. What is $G(1)$?

   (A) 1/3 **(B)** 4/9 **(C)** 13/9 **(D)** 23/9 **(E)** $\boxed{25/9}$

4. Consider the function f defined by:

$$f(x) = \begin{cases} 2x^2 - 3 & \text{if } x < 0 \\ 5x - 3 & \text{if } x \geq 0 \end{cases}$$

   Find the slope of the line tangent to f at the point $(-2, 5)$

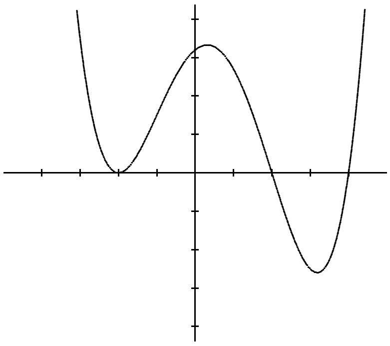
   (A) $\boxed{-8}$ **(B)** -4 **(C)** 0 **(D)** 5 **(E)** 7

5. Let $f(x) = 2x^2 + x$. Evaluate and simplify $\dfrac{f(x+h) - f(x)}{h}$.

   (A) $\boxed{4x + 1 + 2h}$ **(B)** $4x - 2h + h^2$ **(C)** $4x + 2h$

   (D) $4x + 2h + 2$ **(E)** $x^2 + 2h + 2$

6. Questions (a) through (e) refer to the graph of the fourth degree polynomial function f given below.



   (a) The *number* of roots of $f''(x) = 0$ is

   (A) 0 **(B)** 1 **(C)** $\boxed{2}$ **(D)** 3 **(E)** 4

   (b) A good estimate of $f'(-2)$ is

   (A) -1 **(B)** $\boxed{0}$ **(C)** 1 **(D)** 2 **(E)** there is no good estimate

   (c) A good estimate of $f''(-1)$ is

   (A) -1 **(B)** $\boxed{0}$ **(C)** 1 **(D)** 2 **(E)** there is no good estimate

   (d) A good estimate of $f'(2)$ is

   (A) $\boxed{-3}$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

   (e) A good estimate of $f'(0)$ is

   (A) -2 **(B)** -1 **(C)** 0 **(D)** $\boxed{0.8}$ **(E)** 3.2

7. The line tangent to the graph of a function f at the point $(3, 2)$ has y-intercept 8. What is $f'(3)$?

   (A) $\boxed{-2}$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

8. What is the slope of the line tangent to the graph of $f(x) = (2x)^{-2}$ at the point (1,1/4)?

   (A) -2 **(B)** -1 **(C)** $\boxed{-1/2}$ **(D)** $-1/4$ **(E)** $-1/8$

   On all the following questions, **show your work.**
   The next five problems count 10 points each. Compute the following antiderivatives.

9. $\int 6x^2 + x - 1dx$ $\boxed{= 2x^3 + x^2/2 - x + c}$

10. $\int 6x^{\frac{3}{2}} + x^{-\frac{1}{2}} dx$ $\boxed{= 6 \cdot 2/5 \cdot x^{5/2} + 2x^{1/2} + c}$

11. $\int \frac{3x^2 + 2x - 1}{x} dx$ $\boxed{= \int 3x + 2 - 1/x dx = 3x^2/2 + 2x - \ln|x| + c}$

12. $\int \frac{2x + 1}{x^2 + x - 3} dx$ $\boxed{= \ln|x^2 + x - 3| + c}$

13. $\int 2xe^{-x^2} dx$ $\boxed{= -e^{-x^2} + c}$

(45 points) Compute the following integrals.

14. $\int_0^5 2x - 3dx$ $\boxed{= (x^2 - 3x)]_{x=0}^{x=5} = 25 - 15 - 0 = 10}$

15. $\int_1^e \frac{1}{x} dx$ $\boxed{= (\ln|x|)]_{x=1}^{x=e} = \ln e - \ln 1 = 1 - 0 = 1}$

16. $\int_0^1 \frac{e^{-x}}{1 + e^{-x}} dx$ $\boxed{= \ln(|1 + e^{-x}|\Big]_{x=0}^{x=1} = -\ln(1 + e^{-1}) + \ln 2 \approx 0.380}$

17. (20 points) Find the area of the region that is completely enclosed by the graphs of $f(x) = x^2 + 9x - 2$ and $g(x) = 3x - 7$.

The functions agree at the two points $(-5, -22)$ and $(-1, -10)$, and the linear function is larger over the interval $[-5, -1]$, so the integral we want is $\int_{-5}^{-1} (3x-7)-(x^2+9x-2)dx$. Subtracting and antidifferentiating yields $\int_{-5}^{-1} -x^2 - 6x - 5dx = -x^3/3 - 3x^2 - 5x\Big]_{x=-5}^{x=-1} = 10.333.$