Math 1120 **Calculus** **Final Exam**

August 11, 1999 Your name ____________________

1. Suppose the functions f and g are differentiable and their values at certain points are given in the table. The next four problems refer to these functions f and g. Notice that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. Note also that, for example, if $K(x) = f(x) - g(x)$, then $K'(x) = f'(x) - g'(x)$ and $K'(4) = f'(4) - g'(4) = 5 - 10 = -5$. Answer each of the questions below about functions that can be build using f and g.

x	$f(x)$	$f'(x)$
0	2	1
1	2	3
2	5	4
3	1	2
4	3	5
5	6	4
6	0	5
7	4	1

x	$g(x)$	$g'(x)$
0	5	5
1	7	3
2	4	6
3	2	6
4	6	10
5	3	3
6	1	2
7	0	1

(a) The function h is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(3)$. $\boxed{h'(x) = f'(g(x)) \cdot g'(x), \text{ so } h'(3) = f'(g(3)) \cdot g'(3) = f'(2) \cdot 6 = 24.}$

(b) The function k is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(1)$. $\boxed{k'(x) = f'(x)) \cdot g(x) + g'(x) \cdot f(x), \text{ so } k'(1) = 3 \cdot 7 + 3 \cdot 2 = 27.}$

(c) The function H is defined by $H(x) = f(f(x))$. Use the chain rule to find $H'(2)$. $\boxed{H'(x) = f'(f(x)) \cdot g'(x), \text{ so } h'(2) = f'(f(2)) \cdot f'(2) = f'(5) \cdot 4 = 16.}$

(d) Let $Q(x) = f(f(x) - g(x))$. Find $Q'(5)$.

$\boxed{Q'(x) = f'(f(x) - g(x)) \cdot (f'(x) - g'(x)), \text{ so } Q'(5) = f'(6-3) \cdot (4-3) = f'(3) = 2.}$

(e) Find the derivative of the function f/g at the point $x = 4$.

$\boxed{\text{Apply the quotient rule to get} \frac{f'(4)g(4) - g'(4)f(4)}{(g(4))^2} = \frac{30-30}{36} = 0.}$

2. Suppose that the derivative of the function f is given by

$$f'(x) = x^2 - 6x + 5.$$

Note: you are given the *derivative* function! Answer the following questions about f.

(a) Find an interval over which f is increasing.

$f'(x) = (x-1)(x-5)$ so f is montonic on $(-\infty, 1), (1, 5)$, and $(5, \infty)$. Observe that f' is positive over the first and last of these.

(b) Find the location of a relative maximum of f.

$f''(1) = -6 < 0$ implies that f has a relative max at 1.

(c) Find the location of a relative minimum of f.

$f''(5) = 4 > 0$ implies that f has a relative min at 5.

(d) Find an interval over which f is concave upwards.

$f''(x) > 0$ for all $x > 3$ implies that f is concave up on $(3, \infty)$.

(e) Suppose $f(1) = 3$. Find $f(2)$.

$f(x) = x^3/3 - 3x^2 + 5x + c$ for some constant c. Solve $f(1) = 3$ for c to get $c = 2/3$. Then $f(2) = 4/3$.

3. Compute each of the following derivatives.

(a) $\frac{d}{dx}\sqrt{x^3+1}$ $\boxed{\frac{3x^2}{2\sqrt{x^3+1}}}$

(b) $\frac{d}{dx}\ln(x^3+1)$ $\boxed{\frac{3x^2}{x^3+1}}$

(c) Let $f(x) = \dfrac{d}{dx}e^{x^2+1} \cdot e^{2x}$. Find $f'(x)$. $\boxed{f'(x) = 2(x+1)e^{(x+1)^2}}$

(d) $\dfrac{e^x}{x}$ $\boxed{\frac{x-1}{x^2}e^x}$

4. Compute the following antiderivatives.

   (a) $\int 6x^3 - 5x - 1 dx$ $\boxed{3/2 \cdot x^4 - 5/2 \cdot x^2 - x + c}$

   (b) $\int 6x^{\frac{3}{2}} + x^{-\frac{1}{2}} dx$ $\boxed{6 \cdot 2/5 \cdot x^{5/2} + 2x^{1/2} + c}$

   (c) $\int \frac{3x^3 + 2x - 1}{x} dx$ $\boxed{\int 3x + 2 - 1/x dx = 3x^2/2 + 2x - \ln|x| + c}$

   (d) $\int \frac{2x + 1}{x^2 + x - 3} dx$ $\boxed{\ln|x^2 + x - 3| + c}$

5. Compute the following definite integrals.

   (a) $\int_0^2 2xe^{-x^2} dx$ $\boxed{-e^{-x^2}]_0^2 = 1 - e^{-4} \approx 0.9816}$

   (b) $\int_0^5 (2x-1)\sqrt{x^2 - x + 5}\, dx$ $\boxed{2/3(x^2 - x + 5)^{3/2}]_0^5 = \frac{10}{3}(25 - \sqrt{5}) \approx 75.8798}$

6. Find the largest interval over which $f(x) = 4x^3 + 39x^2 - 42x$ is decreasing.

   $f'(x) = 12x^2 + 78x - 42$, so the critical points are $x = -7$ and $x = 1/2$. Use the test interval method to find that $f'(x) < 0$ on the interval $(-7, 1/2)$, so f is decreasing over that interval.

7. Find a function $G(x)$ whose derivative is $3x^2 - 7$ and for which $G(4) = 9$.

8. Find the area of the region bounded by $y = x^{3/2}$, the x-axis, and the lines $x = 0$ and $x = 4$.

9. Find the area of the region caught between the graphs of the functions

$$f(x) = -x^2 + 4x \text{ and } g(x) = -2x + 5.$$

10. An apartment complex has 100 two-bedroom units for rent all at the same price. The monthly profit from renting x units is given by

$$P(x) = -10x^2 + 1760x - 50000$$

dollars. Find the number of units that should be rented out to maximize the profit. What is the maximum monthly profit realizable?