Math 1120 **Calculus** **Section 6** **Final Exam**

Thursday, Dec. 16, 1999 Your name _____

The multiple choice problems count five points each.

1. Let $f(x) = 2x^3 - 3x + 8$. What is $f'(1)$?

   (A) 0 **(B)** 3 **(C)** 8 **(D)** 12 **(E)** 21

2. Let $h(x) = e^{x^2}$. What is $h''(0)$?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

3. Let $g(x) = \ln(1 + \frac{1}{x})$. What is the slope of the line tangent to the graph of g at the point $(2, \ln 1.5)$?

   (A) $-1/3$ **(B)** $-1/6$ **(C)** $-1/12$ **(D)** $1/12$ **(E)** $2/3$

4. The distance from the point $(3, 4)$ to the point $(-1, x)$ is 5. Which of the following could be x?

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 8

5. What is $\lim_{h \to 0} \dfrac{\sqrt{1+2h} - 1}{h}$?

   (A) -1 **(B)** 0 **(C)** $1/2$ **(D)** 1 **(E)** 2

6. The slope of the line that contains the points $(-1, y)$ and $(4, -12)$ is -2? What is y?

   (A) -3 **(B)** -2 **(C)** 3 **(D)** 5 **(E)** 6.2

7. At which of the following points is the second derivative of

   $$x^4 - 6x^3 + 12x^2 + 2x + 2$$

   negative?

   (A) $-1/2$ **(B)** $1/2$ **(C)** $3/2$ **(D)** $5/2$ **(E)** $7/2$

8. What is the slope of the line perpendicular to the line $2y + x = 6$?

   (A) -3 **(B)** -2 **(C)** $-1/2$ **(D)** 2 **(E)** 3

9. The function f has second derivative given by $f''(x) = 2x-1$, and also satisfies $f(0) = 19/6$ and $f'(0) = 1$. What is $f(1)$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. Suppose $f'(x) = 2x^2$ and $g(x) = 3x - 1$. What is $\frac{d}{dx}(f \circ g(x))$?

    (A) $2(3x-1)^2$ **(B)** $3(9x^2-6x+1)$ **(C)** $6(9x^2-6x+1)$

    (D) $2x^2(3x-1)$ **(E)** $4x(3x-1)$

11. Let

$$f(x) = \begin{cases} 2+\sqrt{1-x} & \text{if } x \leq 1 \\ 1/(1-x) & \text{if } x > 1 \end{cases}$$

    and let $g(x) = 2x - 1$. Compute $g(f(2) - f(g(1))$.

    (A) -7 **(B)** -5 **(C)** -1 **(D)** 1 **(E)** 5

12. What is $\lim_{h\to 0} \dfrac{\frac{2}{3+h} - \frac{2}{3}}{h}$?

    (A) $-2/3$ **(B)** $-2/9$ **(C)** $2/9$ **(D)** $2/3$ **(E)** $3/2$

13. What is the number of vertical asymptotes of the function h defined by

$$h(x) = \frac{(x^2-1)(x^2-4)}{(x-3)(x-2)(x-1)(x)(x+1)(x+2)^2}?$$

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. It takes exactly 12 years for $\$P$ invested at an annual rate r compounded continuously to triple. What is r (to the nearest 0.001)?

    (A) 0.075 **(B)** 0.080 **(C)** 0.086 **(D)** 0.092 **(E)** 0.102

15. What is $(2x-3)\cdot(x-1)-(2x-3)\cdot x-1$?

    (A) 0 **(B)** $2-2x$ **(C)** $2x-4$ **(D)** $2x-3$ **(E)** $2x-2$

16. The number x satisfies $2^x=5$. What is 7^x?

    (A) 90.19 **(B)** 90.83 **(C)** 91.09 **(D)** 91.55 **(E)** 91.68

17. Suppose f is a continuous function such that $f(0)=-1, f(1)=2$, $f(2)=-3$, $f(3)=4$, $f(4)=-2$, and $f(5)=-3$. What is the fewest number of zeros f could have?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

18. Suppose the function

$$f(x)=\begin{cases} x+2 & \text{if } x\le 2 \\ kx-6 & \text{if } x>2 \end{cases}$$

    is continuous at $x=2$. Then $k=$

    (A) 1 **(B)** 2 **(C)** 5 **(D)** 6 **(E)** 7

19. It takes 10 years for a \$1000 invested at an annual rate of r compounded quarterly to double. What is r?

    (A) 0.070 **(B)** 0.072 **(C)** 0.074 **(D)** 0.076 **(E)** 0.078

20. What is $\int_0^3 x^2+2x+1dx$?

    (A) 13 **(B)** 17 **(C)** 21 **(D)** 25 **(E)** 27

21. (20 points) Compute each of the following derivatives.

(a) $\frac{d}{dx}\sqrt{x^3+1}$

(b) $\frac{d}{dx}\ln(2x^3+1)$

(c) Let $f(x) = e^{x^2} \cdot e^{-2x+1}$. Find $f'(x)$.

(d) $\dfrac{d}{dx}\dfrac{e^{2x}}{x}$

22. (20 points) Compute the following antiderivatives.

    (a) $\displaystyle\int \frac{3x^2}{2\sqrt{x^3+1}}dx$

    (b) $\displaystyle\int \frac{x^3-2x-1}{x}dx$

    (c) $\displaystyle\int \frac{3x^2+1}{x^3+x-3}\,dx$
    Hint: Let $u = x^3 + x - 3$.

23. (16 points) Compute the following integrals.

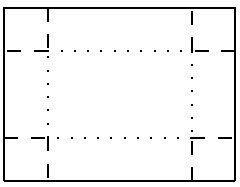
    (a) $\displaystyle\int_0^2 2xe^{-x^2}dx$

    (b) $\displaystyle\int_0^5 (2x-1)\sqrt{x^2-x+5}\,dx$

24. (10 points) Find a function $G(x)$ whose derivative is $1/(x-5)$ and whose value at $x = 6$ is 9.

25. (10 points) Find the area of the region bounded by $y = x^{3/2}$, the x-axis, and the lines $x = 0$ and $x = 4$.

26. (20 points) A 16in. by 12in. sheet of paper is used to build a topless box as follows: an x-in. by x-in. square is cut from each corner, and the resulting rectangular pieces are folded upward along the dotted lines to form the sides of the box.



    (a) What is the volume V of the resulting box?

    (b) Find $\frac{d}{dx}V(x)$.

    (c) What is the domain of V?

    (d) Find all stationary points of V.

    (e) What value of x maximizes the volume?

    (f) What is the maximal volume?

27. (20 points) According to Newton's Law of Cooling, the temperature $F(t)$ of a body in a surrounding medium changes at a rate that is proportional to the difference between the temperature of the body and the temperature of the surroundings. It follows that $F(t) = T + Ae^{-kt}$, where t is expressed in minutes, T is the temperature in Celcius of the surrounding medium, and A and k are constants. A hard-boiled egg at 98°C is put in a pan under running 10°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. How much longer will it take the egg to reach 20°C? Use the following steps to solve the problem. Show your work in detail.

    (a) What is T?

    (b) Use the fact that $f(0) = 98°$ and the value of T to find A.

    (c) Use the values of T and A and the temperature of the egg after five minutes to find the value of k.

    (d) Use the values of A, T, and k to find the time required for the egg to become 20°C.

Solutions and answers.

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    (c) Let $f(x)=e^{x^2}\cdot e^{-2x+1}$. Find $f'(x)$. $\boxed{2(x-1)e^{(x-1)^2}}$

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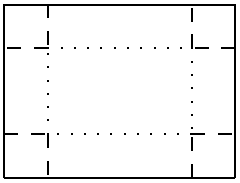

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