

May 4, 2001

Name \_\_\_\_\_

The first five problems count 7 points each (total 35 points) and rest count as marked. There are 195 points available. Good luck.

1. Consider the function  $f$  defined by:

$$f(x) = \begin{cases} 2x^2 - 3 & \text{if } x < 0 \\ 5x - 3 & \text{if } x \geq 0 \end{cases}$$

Find the slope of the line which goes through the points  $(-2, f(-2))$  and  $(3, f(3))$ .

- (A)  $7/5$    (B) 2   (C)  $17/5$    (D) 5   (E) 7

2. The distance between the point  $(6.5, 8.5)$  and the midpoint of the segment joining the points  $(2, 3)$  and  $(5, 6)$  is

- (A)  $\sqrt{22}$    (B)  $\sqrt{23}$    (C) 5   (D)  $\sqrt{26}$    (E) 6

3. Let  $f(x) = 2x + 3$  and  $g(x) = 3x - 9$ . Which of the following does not belong to the domain of  $f/g$ ?

- (A) 1   (B) 3   (C) 6   (D) 9   (E) The domain of  $f/g$  is the set of all real numbers.

4. The line tangent to the graph of a function  $f$  at the point  $(2, 5)$  on the graph also goes through the point  $(0, 7)$ . What is  $f'(2)$ ?

- (A) -2   (B) -1   (C) 0   (D) 1   (E) 2

5. What is the slope of the tangent line to the graph of  $f(x) = x^{-2}$  at the point  $(2, 1/4)$ ?

- (A)  $-1/4$    (B)  $-1/8$    (C)  $-1/16$    (D)  $-1/256$    (E)  $-1/512$

6. (15 points) Let  $f(x) = 1/(3x)$ .
- Construct  $\frac{f(2+h)-f(2)}{h}$
  - Simplify and take the limit of the expression in (a) as  $h$  approaches 0 to find  $f'(2)$ .
  - Use the information found in (b) to find an equation for the line tangent to the graph of  $f$  at the point  $(2, 1/6)$ .
7. (10 points) Find the rate of change of  $f(t) = e^{2t} \cdot \ln(t)$  when  $t = 1$ .
8. (20 points) Suppose the functions  $f$  and  $g$  are differentiable and their values at certain points are given in the table. The next four problems refer to these functions  $f$  and  $g$ . Notice that, for example, the entry 1 in the first row and third column means that  $f'(0) = 1$ . Note also that, for example, if  $K(x) = f(x) - g(x)$ , then  $K'(x) = f'(x) - g'(x)$  and  $K'(4) = f'(4) - g'(4) = 5 - 10 = -5$ . Answer each of the questions below about functions that can be build using  $f$  and  $g$ .

$x$	$f(x)$	$f'(x)$	$x$	$g(x)$	$g'(x)$
0	2	1	0	5	5
1	2	3	1	7	3
2	5	4	2	4	6
3	1	2	3	2	6
4	3	5	4	6	10
5	6	4	5	3	3
6	0	5	6	1	2
7	4	1	7	0	1

- The function  $h$  is defined by  $h(x) = f(g(x))$ . Use the chain rule to find  $h'(3)$ .
- The function  $k$  is defined by  $k(x) = f(x) \cdot g(x)$ . Use the product rule to find  $k'(1)$ .
- The function  $H$  is defined by  $H(x) = f(f(x))$ . Use the chain rule to find  $H'(2)$ .
- Let  $Q(x) = f(f(x) - g(x))$ . Find  $Q'(5)$ .

9. (10 points) A radioactive substance has a half-life of 27 years. Find an expression for the amount of the substance at time  $t$  if 20 grams were present initially.

10. (10 points) If  $h = g \circ f$  and  $f(1) = 2, g'(2) = 5, f'(1) = -3$  find  $h'(1)$ .

11. (15 points) Let  $f(x) = x^4 + 2x^3 - 6x^2 + x - 5$ .

(a) Find the interval(s) where  $f$  is concave upward.

(b) Find the inflection points of  $f$ , if there are any.

12. (20 points) A ball is thrown upwards from the top of a building that is 200 feet tall. The position of the ball at time  $t$  is given by  $s(t) = -16t^2 + 36t + 200$ , where  $s(t)$  is measured in feet and  $t$  is measured in seconds.

(a) What is the velocity of the ball at time  $t = 0$ ?

(b) What is the velocity of the ball at time  $t = 1$ ?

(c) How many seconds elapse before the ball hits the ground?

(d) What is the speed of the ball when it hits the ground?

(e) What is the acceleration of the ball at the time it hits the ground?

13. (20 points)

- (a) Let  $f(x) = 2x^2$  and compute the Riemann sum of  $f$  over the interval  $[1, 9]$  using four subintervals of equal length ( $n = 4$ ) and choosing the representative point in each subinterval to be the midpoint of the subinterval.

(b) Compute

$$\int_1^9 2x^2 dx$$

and compare this value with the one in part a.

14. (10 points) Find an equation for the line tangent to the graph of  $f(x) = x \ln(x) - x$  at the point  $(1, f(1))$ .

15. (10 points) Evaluate  $\int 3x^2\sqrt{x^3+1} dx$

16. (10 points) Evaluate  $\int_1^3 x^3 \cdot (x^4 - 2)^2 dx$

17. (10 points) Evaluate  $\int_0^4 2xe^{x^2} dx$