Math 1120, Section 1 **Calculus** **Final Exam**

May 7, 2014 **Name** ______________________________

Each of the first 17 problems are worth 10 points. The other problems are marked. The total number of points available is 285. Throughout the free response part of this test, to get credit you must **show your work.**

1. Let $f(x) = x^3 - 2x + 4$. Find an equation for the line tangent to the graph of f at the point $(1, f(1))$.

2. Consider the function $f(x) = xe^{2x}$. Find a critical point. Does f have a relative maximum, a relative minimum, or neither at that point?

3. What is $\lim_{x\to\infty} \dfrac{(x-2)(2x-3)}{(3x+2)(4x-1)}$?

4. What is $\lim_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$?

5. What is $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x^2 - 16}$?

6. Let $F(x)$ be an antiderivative of $3x^2 - 2x$. What is the growth of $F(x)$ over the interval $[1, 5]$?

7. Let $H(x) = \ln(4x^2 + 12x + 10) - 2x$. Find a critical point.

8. Let $g(x) = 2x^3 - 7x^2 + 4x - 10$. Find the intervals over which g is increasing?

9. Let $k(x) = 2x^4 - 14x^3 + 30x^2 + 10x$. Find the intervals over which k is concave upwards?

10. What is the value of $\int_2^5 \frac{d(2x-5)^4}{dx}\ dx$?

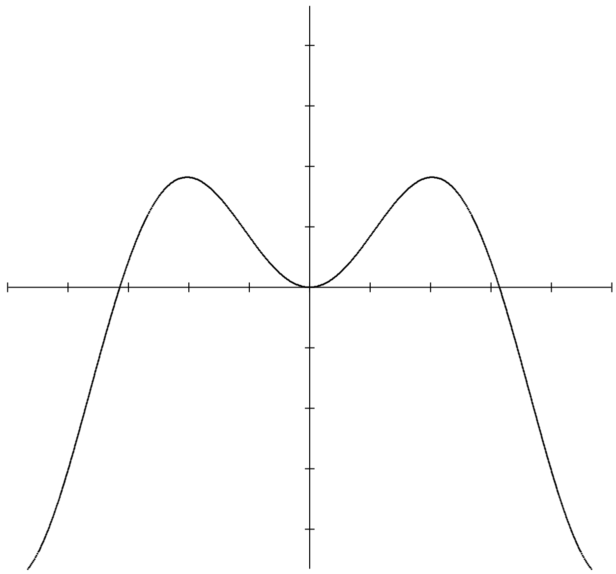
11. Find all of the zeros of the polynomial $p(x) = (x-1)^3(x+2)^2 - 4(x-1)^2(x+2)$.

12. How many solutions does the equation $|x^2 - 4| = 3$ have? Find them all.

13. Let $h(x) = x^3(x^2 - 1)^2$. Find all the critical points of h.

14. Find a value of t that satisfies $\int_{-t}^{t} x^3 + x^2 \, dx = 18$.

15. The next three problems may have multiple correct answers. Circle **all** the correct answers. The three problems all refer to the graph of the function $g'(x)$ given below.



(a) The function g has a relative maximum or minimum

(A) near -3 **(B)** at $x = -2$ **(C)** at $x = 0$

(D) at $x = 2$ **(E)** near 3

(b) The function g has an inflection point (ie, a change in concavity)

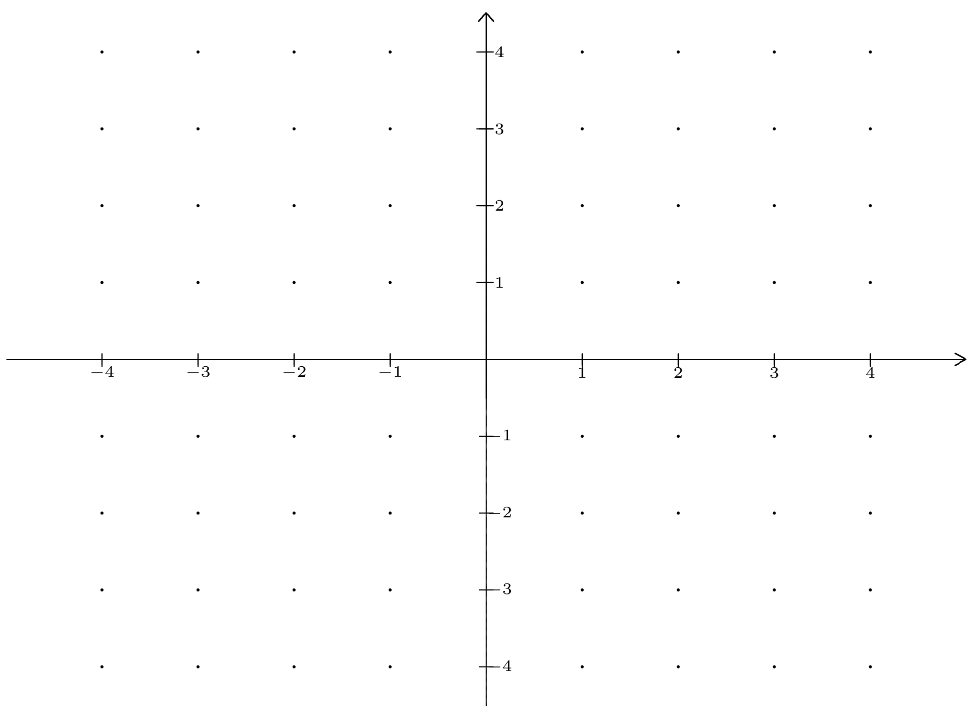
(A) near -3 **(B)** at $x = -2$ **(C)** at $x = 0$

(D) at $x = 2$ **(E)** near 3

(c) The function g increases over

(A) $[-5, -2]$ **(B)** $[-3, 0]$ **(C)** $[0, 3]$ **(D)** $[-2, 2]$ **(E)** $[2, 3]$

16. (25 points) Consider the rational function $r(x) = \frac{(2x-4)(x^2-9)(x+3)}{(x^2-4)(x+1)(x)}$. Find the vertical and horizontal asymptotes, and the zeros of the function. Build the sign chart for the function. Sketch the graph on the coordinate system provided. Use the graph to state the intervals over which the function is increasing. Using the graph you find, build the sigh chart for the derivative function $r'(x)$.



17. (20 points) Find the area caught between the graphs of the two functions $f(x) = -(x-1)(x-3)$ and $g(x) = x - 3$.

18. (20 points) Among all the lines tangent to the graph of $f(x) = 6x^2 - x^3$ over the interval $(0, 6)$, which one has the greatest slope. Find an equation for the one with the greatest slope.

19. (30 points) Consider the curve $y = x^2$, your favorite parabola. In this problem you are asked to find the point on the curve that is closest to the point $(18, 0)$. In case you missed this problem when it appeared on set 8, the steps below will enable you to solve it.

    (a) Note that $(0, 0)$ belongs to the curve and the distance from $(18, 0)$ to $(0, 0)$ is 18. Find two other points on the curve and compute their distance from $(18, 0)$.

    (b) Note that all the points on the curve are of the form (x, x^2) for some real number x. Suppose the point on the curve closest to $(18, 0)$ is (a, a^2). What is the slope, in terms of a, of the line tangent to the curve at the point (a, a^2)?

    (c) What is the slope, in terms of a, of the line connecting (a, a^2) with $(18, 0)$. There are two ways to get this slope. One way to find a is to equate these two slopes.

(d) Build the function $D(x)$ whose value at x is the distance from $(18, 0)$ to the point (x, x^2) on the curve. Then, as we did in class, find the derivative of the function $(D(x)^2$. Evaluate your derivative at each of the points $x = 0, x = 1$, and $x = 2$.

(e) Find the distance from $(18, 0)$ to the nearest point on the curve.

20. (20 points) Amber Airlines runs chartered flights to Costa Rica. They expect 300 passengers and they charge each passenger \$200. However, if more than 300 persons sign up for the flight, they agree to charge \$0.50 less per ticket for each extra person. For example, if 302 passengers sign up, the airline charges each of the 302 passengers \$199.00.

(a) Note that if $x = 300$, the revenue is $300 \cdot \$200 = \$60000$. Build the revenue function $R(x)$, where x represents the total number of passengers.

(b) How many passengers result in the maximum revenue?

(c) What is that maximum revenue?