May 6 2015 **Name** ______________________________

The total number of points available is 260. Throughout this test, show your work. Throughout this test, you are expected to use calculus to solve problems. Circle your answers in each of the multiple choice problems. Some of the multiple choice problems have **more than one** correct answer. Circle all the correct choices. Please note, this practice test is an amalgamation or the two tests given in sections 1 and 2.

1. In the year 2000 the population of the earth was estimated to be 6.5 billion people. Today the estimate is 7.3 billion. Which of the following is closest to the annual population growth?

   (A) 1% **(B)** 2% **(C)** 3% **(D)** 4% **(E)** 5%

2. Which of the following is closest to the number of years that it takes for a continuously compound investment at 8% to triple in value?

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

3. Which of the following is closest to the area caught between the x-axis and the graph of $y = x(4 - x)$?

   (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

4. A cubic polynomial has zeros at $x = 0, x = 1$, and $x = 3$. The coefficient of the x^3 term is 1. Which of the following is closest to the area of the region bounded by the graph of the function over the interval $[0, 1]$?

   (A) 1/4 **(B)** 1/3 **(C)** 5/12 **(D)** 1/2 **(E)** 7/12

5. Which of the following is closest to the area of the region bounded by the lines $x = 0, x = 2$, the x-axis and the graph of the function $\frac{d}{dx}e^{x^2}$?

   (A) 50.2 **(B)** 51.7 **(C)** 53.6 **(D)** 54.6 **(E)** 55.2

6. The number $|3\pi - 10 - \sqrt{2}| + |4 - \sqrt{2}|$ belongs to one of the intervals listed. Which one is it?

   (A) $[2, 3)$ **(B)** $[3, 4)$ **(C)** $[4, 5)$ **(D)** $[5, 11)$ **(E)** $[11, 30)$

7. Let $A = (1, 2)$ and $B = (4, 6)$ be two points in the plane. Which of the following equations represents a circle with the segment AB as a radius?

   (A) $(x-1)^2 + (y-2)^2 = 5^2$ **(B)** $(x-1)^2 + (y-2)^2 = 5$

   (C) $(x-4)^2 + (y-6)^2 = 5^2$ **(D)** $(x-1)^2 + (y-6)^2 = 5$

   (E) $(x-4)^2 + (y-2)^2 = 5^2$

8. Which of the following is a critical point of the function $h(x) = \sqrt{x^2+1} - x/2$?

   (A) $\sqrt{1/2}$ **(B)** $\sqrt{1/3}$ **(C)** $\sqrt{1/4}$ **(D)** $\sqrt{1/5}$ **(E)** $\sqrt{1/6}$

9. Which of the following is the slope of the line tangent to the graph of $f(x) = 2x^2 - 3x - 5$ at the point $(1, f(1))$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** None of the above

10. Which of the following is the slope of the line tangent to the graph of $f(x) = xe^x$ at the point $(1, f(1))$?

    (A) e **(B)** $2e$ **(C)** e^2 **(D)** $2e^2$ **(E)** None of the above

11. Which of the following is the slope of the line tangent to the graph of $f(x) = \ln(x^2+2)$ at the point $(1, f(1))$?

    (A) 1/3 **(B)** 2/3 **(C)** 1 **(D)** 4/3 **(E)** None of the above

12. At which of the following points is the function $g(x) = e^{x^3-6x^2}$ decreasing?

    (A) -1 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 7

13. Recall that $\ln(x)$ is defined precisely when $x > 0$. Which of the following points **does not belong** to the domain of the function $g(x) = \ln(2x^2 - 8)$?

    (A) -3 **(B)** -1 **(C)** 1 **(D)** 3 **(E)** 5

14. Which of the following points **does not belong** to the domain of the function $g(x) = \sqrt{|x-3| - 3}$?

    (A) -1 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 7

15. Which of the following points belongs to the domain of the function $g(x) = \frac{1}{(x^2-1)(x^2-4)}$?

    (A) -2 **(B)** -1 **(C)** 0 **(D)** 2 **(E)** 4

16. How many different real number critical points does the function $h(x) = x^2(x^2-4)^3$ have?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

17. Which of the following is a horizontal asymptote of the function $r(x) = \frac{(2x-3)(3x+5)^3}{(9x-1)(x^3-27)}$?

    (A) $y = 1$ **(B)** $y = 2$ **(C)** $y = 3$ **(D)** $y = 6$ **(E)** $y = 0$

18. Which of the following is equal to $\lim\limits_{x\to 1} \dfrac{x^2+6x-7}{x^2+2x-3}$?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. Let $L = \lim\limits_{x\to -3} \dfrac{x^3+27}{x+3}$. Then L belongs to which interval below?

    (A) $(-30, 0)$ **(B)** $(0, 10)$ **(C)** $(10, 20)$ **(D)** $(20, 30)$ **(E)** $(30, 40)$

20. What is the y-intercept of the line tangent to the graph of $f(x) = x^3 - x^2$ at the point $(2, f(2))$?

    (A) -15 **(B)** -12 **(C)** 4 **(D)** 12 **(E)** 20

21. $\lim\limits_{x\to 2} \dfrac{\frac{1}{x} - \frac{1}{3}}{x-3}$ belongs to one of the intervals listed. Which one?

    (A) $[-1, 0)$ **(B)** $[0, 1)$ **(C)** $[1.2)$ **(D)** $[2, 4)$ **(E)** $[4, 10)$

22. $\lim\limits_{x\to 7} \dfrac{\sqrt{x-3}-2}{x-7}$ belongs to which one of the intervals below?

    (A) $(0, 1/5)$ **(B)** $(1/5, 2/5)$ **(C)** $(2/5, 3/5)$

    (D) $(3/5, 4/5)$ **(E)** $(4/5, 1)$

23. $\lim\limits_{x\to -1} \dfrac{x^2-1}{x^3+1}$ belongs to which one of the intervals below?

    (A) $(-1, -4/5)$ **(B)** $(-4/5, -3/5)$ **(C)** $(-3/5, -2/5)$

    (D) $(-2/5, -1/5)$ **(E)** $(0, 1)$

24. $\lim_{x\to\infty} \dfrac{(2x-3)^3}{11-x^3}$ belongs to which of the intervals listed?

(A) $(-10,-3)$ **(B)** $(-3,0)$ **(C)** $(0,3)$ **(D)** $(3,10)$ **(E)** the limit does not exist

25. The function f is defined throughout $[-1,1]$. Suppose $lim_{x\to 0} f(x) = 3$, what is $f(0)$?

(A) 0 **(B)** 3 **(C)** It must be close to 3

(D) $f(0)$ is not defined **(E)** Not enough information is given

26. If a function is always positive, then what must be true about its derivative?

(A) The derivative is always positive.

(B) The derivative is never negative.

(C) The derivative is increasing.

(D) The derivative is decreasing.

(E) You can't conclude anything about the derivative.

27. If a function is increasing over the whole number line, then what must be true about its derivative?

(A) The derivative is never positive.

(B) The derivative is never negative.

(C) The derivative is increasing.

(D) The derivative is decreasing.

(E) You can't conclude anything about the derivative.

28. How many lines L satisfy both

- L is tangent to the graph of $f(x) = x^2$, and
- L passes through the point $(0,-2)$.

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

29. What is the y-intercept of the line tangent to the graph of $f(x) = (1 + e^{-2x+4})^2$ at the point $(2, f(2))$.

(A) -8 **(B)** 8 **(C)** 12 **(D)** 15 **(E)** 20

30. What is $\lim_{h\to 0} \dfrac{\sqrt{(1+h)^2+3}-2}{h}$?

(A) 1/4 **(B)** 1/2 **(C)** 1 **(D)** 2 **(E)** does not exist

31. If $f(x) = (x^2 + \ln(x))^2$, what is $f'(1)$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

32. A radioactive substance has a half-life of 29 years. If 30 grams were present initially, how many grams remain after 80 years?

(A) 3.2 grams **(B)** 4.4 grams **(C)** 5.1 grams

(D) 6.3 grams **(E)** 8.0 grams

33. For what value(s) of x is the line tangent to $y = 4 - x^3$ parallel to the line $y = -12x + 17$.

(A) -2 **(B)** -1 **(C)** 0 **(D)** 2 **(E)** 4

34. What is $\lim_{x\to 1} \dfrac{x^2-1}{x^2-2x+1}$?

(A) 0 **(B)** 1 **(C)** -1 **(D)** 2 **(E)** does not exist

35. Let $h(x) = e^{x^2}$, What is $h''(0)$?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

36. Let $g(x) = \ln(1 + \frac{1}{x})$. What is the slope of the line tangent to the graph of g at the point $(2, \ln(1.5))$?

    (A) $-1/3$ **(B)** $-1/6$ **(C)** $-1/12$ **(D)** $1/12$ **(E)** $2/3$

37. The distance from the point $(3, 4)$ to the point $(-1, x)$ is 5. Which of the following could be x?

    (A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

38. At which of the following points is function $y = x^4 - 6x^3 + 12x^2 + 2x + 5$ concave downwards?

    (A) $-1/2$ **(B)** $1/2$ **(C)** $3/2$ **(D)** $5/2$ **(E)** $7/2$

39. What is the slope of the line perpendicular to the line $2y + x = 6$?

    (A) -3 **(B)** -2 **(C)** $-1/2$ **(D)** 2 **(E)** 3

40. The function$f$ has second derivative given by $f''(x) = 2x - 1$, and also satisfies $f(0) = 19/6$ and $f'(0) = 1$. What is $f(1)$?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

41. What is the number of vertical asymptotes of the rational function

$$r(x) = \frac{(x^2 - 1)(x + 2)(x^2 - 16)}{(x - 3)x(x - 1)(x + 1)(x^2 - 4)}?$$

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

42. Let

$$f(x) = \begin{cases} 2 + \sqrt{1 - x} & \text{if } x \leq 1 \\ 1/(1 - x) & \text{if } x > 1 \end{cases}$$

    and $g(x) = 2x - 1$. Compute $g(f(2)) - f(g(1))$.

    (A) -7 **(B)** -5 **(C)** -1 **(D)** 1 **(E)** 5

43. What is the value of $(2x-3)\cdot(x-1)-(2x-3)\cdot x-1$?

    (A) 0 **(B)** $2-2x$ **(C)** $2x-4$ **(D)** $2x-3$ **(E)** $2x-2$

44. The number x satisfies $2^x = 7$. What is 5^x?

    (A) 90.2 **(B)** 90.8 **(C)** 91.1 **(D)** 91.3 **(E)** 91.7

45. Suppose f is a continuous function such that $f(0) = -1, f(1) = 2, f(2) = -3, f(3) = 4, f(4) = -2$, and $f(5) = -4$. What is the fewest number of zeros f could have?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

46. Which of the following is closest to $\displaystyle\int_e^3 \frac{1}{x}\,dx$

    (A) 0 **(B)** 0.1 **(C)** 0.2 **(D)** 0.3 **(E)** 0.4

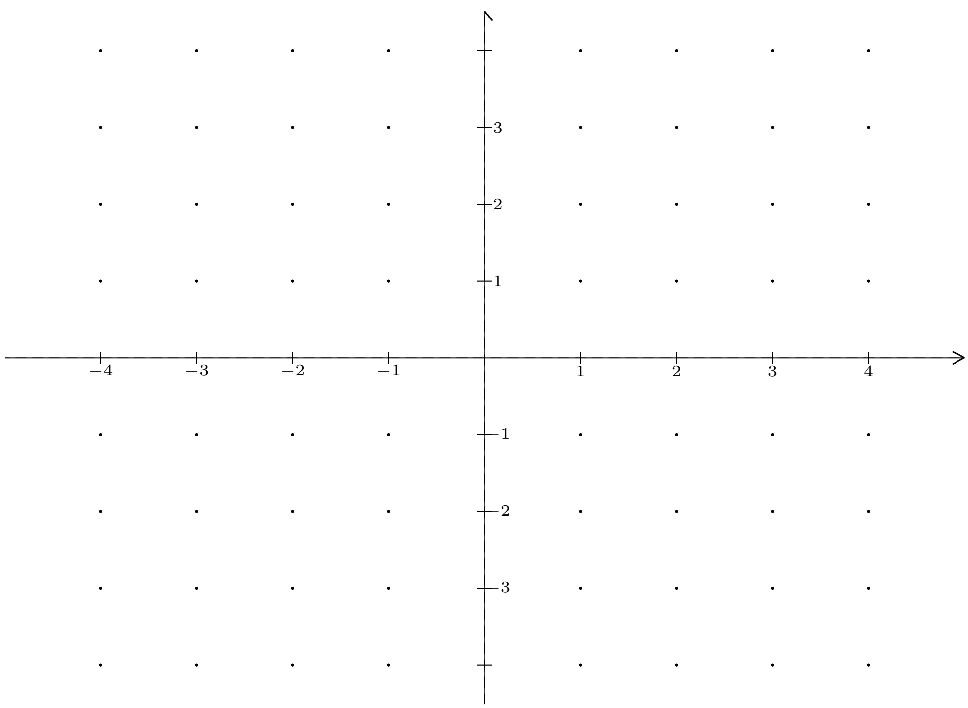
47. Which of the following is closest to $\displaystyle\int_0^5 \frac{2x}{x^2+1}\,dx$

    (A) 2.56 **(B)** 3.17 **(C)** 3.26 **(D)** 3.90 **(E)** 4.11

48. Which of the following is closest to $\displaystyle\int_0^1 2x(x^2+1)^4\,dx$

    (A) 5.1 **(B)** 5.7 **(C)** 6.2 **(D)** 6.8 **(E)** 7.1

49. (30 points) A rational function $r(x)$ has zeros at $x = -2$ and $x = 0$, vertical asymptotes at $x = 1$ and $x = 3$, and a horizontal asymptote at $y = -2$. Find a symbolic representation of such a function, and build its sign chart. Sketch the graph below. Estimate the location of a critical point of r.



50. (10 points) The annual rate of growth of the population of the world is declining. What does that mean in terms discussed in this course?

51. (10 points) Compute $\int_0^1 (x-1)^6 x^2 \, dx$. Show your work! If you leave your answer as a decimal approximation, be sure to write the fraction difference that it approximates. IE. do not rely on your calculator.

52. (30 points) Consider the function $f(x) = x^2(x^2-4)^3$. Find the intervals over which f is increasing. Show all your work including your calculation of $f'(x)$.

53. (15 points) The percentage of alcohol in a person's bloodstream t hours after drinking 4 fluid ounces of whiskey is given by

$$A(t) = 0.24te^{-0.3t}, \qquad 0 \leq t \leq 6.$$

(a) How fast is the percentage of alcohol in the person's bloodstream changing after 1 hour?

(b) At what time is the percentage maximized?

(c) What is that maximum percentage?

54. (30 points) Let $A = (6, 6)$, $B = (1, 4)$, $C = (1, 0)$ and $D = (6, 0)$ be the vertices of a quadrilateral in the plane.

   (a) Sketch the figure and use geometry to find the area of $ABCD$

   (b) Find an equation for the linear function (the line) that goes through the points C and D. Give this function the name f.

   (c) Use calculus to find the area of the region R defined as follows:

$$R = \{(x, y) : 1 \leq x \leq 6,\ 0 \leq y \leq f(x)\}$$