

December 11, 2001

Your name _____

The value of each problem is shown. You must show your work on the problems. The total number of points available is 246.

1. (8 points) Consider the function f defined by:

$$f(x) = \begin{cases} 2x^2 - 3 & \text{if } x < 0 \\ 5x^2 - 3x & \text{if } x \geq 0 \end{cases}$$

Find an equation for the line that is tangent to the graph of f at the point $(3, f(3))$.

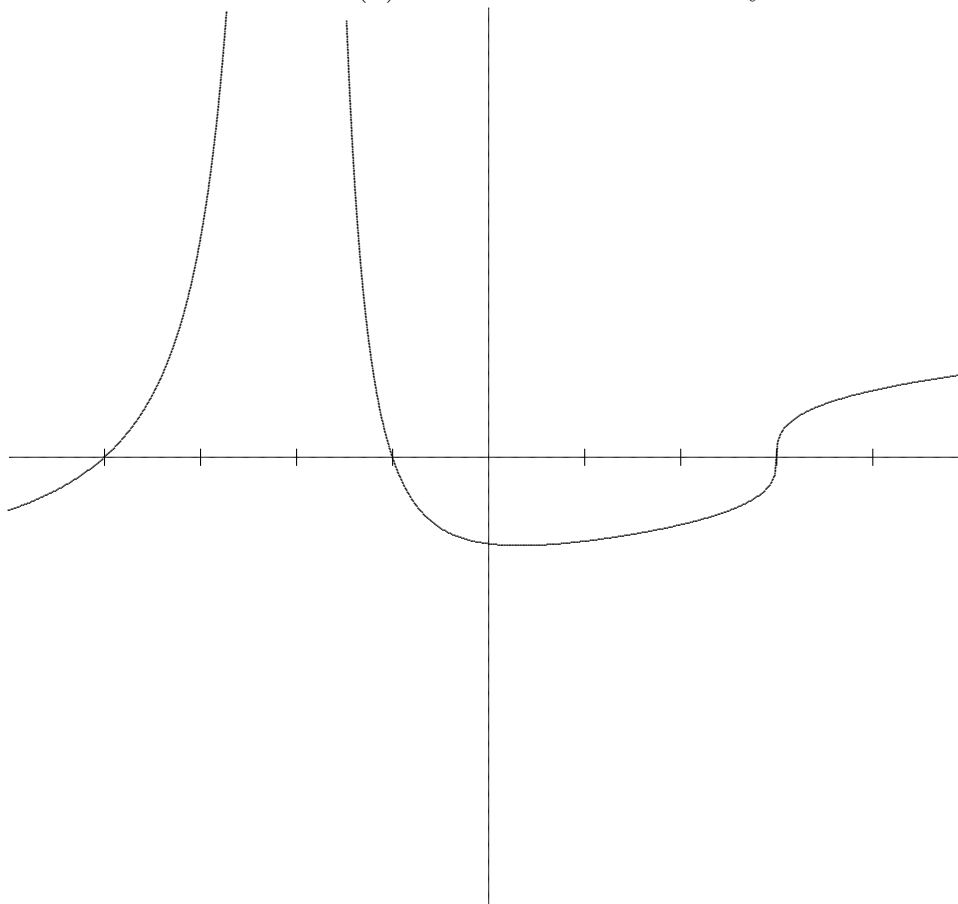
2. (8 points) Again referring to the function f defined in problem 1, what is the slope of the line joining the points $(-2, f(-2))$ and $(2, f(2))$?
3. (8 points) What is the distance between the point $(6, 8)$ and the midpoint of the segment joining the points $(2, 3)$ and $(10, -7)$?
4. (8 points) Find an equation for each horizontal asymptote of $r(x)$?

$$r(x) = \frac{(x+4)(x^2-1)(3x^2-4)}{(x^2+x-12)(x-1)^4}$$

5. (8 points) Referring again to the function $r(x)$ in the previous problem, find an equation for each vertical asymptote of $r(x)$?

6. (8 points) The line tangent to the graph of a function f at the point $(1, 5)$ on the graph also goes through the point $(0, 8)$. What is $f'(1)$?
7. (8 points) What is the slope of the tangent line to the graph of $f(x) = e^{2x}$ at the point $(1, e^2)$?
8. (10 points) Let $g(x) = (2x - 3)^2(x + 1)^2$. Find $g'(x)$ and the critical points of g . Express g' in factored form.
9. (10 points) Let $f(x)$ be the function defined on $[-3, 4]$ by the equation $f(x) = x^3 - 9x + 4$. Find the absolute maximum and absolute minimum of f and the locations where those extrema occur.
10. (15 points) Suppose $f''(x) = (x - 5)(2x + 3)(x - 3)(2x + 9)$. Find the intervals over which f is concave upwards. Note that the second derivative has already been found for you.

11. (10 points) The graph of a function $G(x)$ is shown below. Sketch the graph of the derivative function $G'(x)$ on the same coordinate system.



12. (40 points) Compute the following antiderivatives.

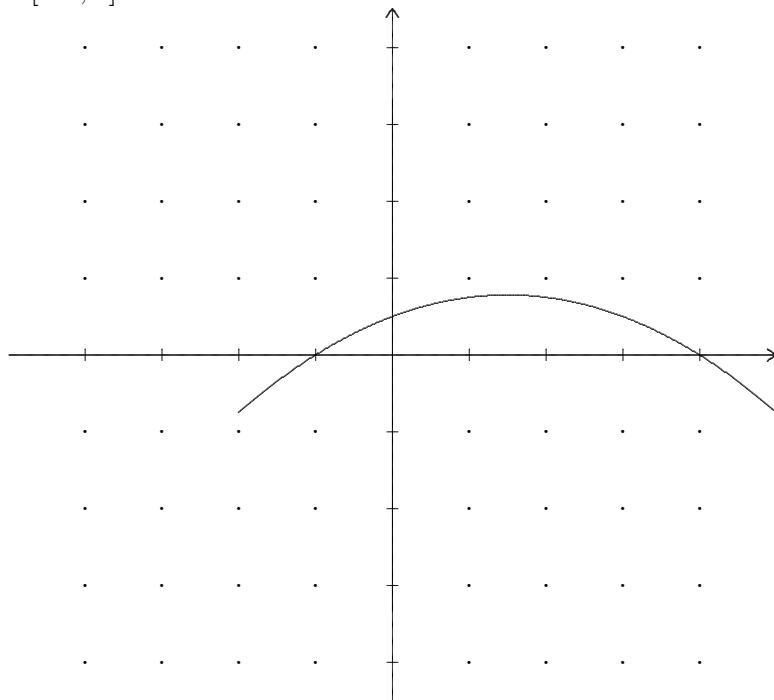
(a) $\int 6x^3 - 5x - 1 dx$

(b) $\int 6x^{\frac{3}{2}} + x^{-\frac{1}{2}} dx$

(c) $\int \frac{3x^3 + 2x - 1}{x} dx$

(d) $\int \frac{2x + 1}{x^2 + x - 3} dx$

13. (10 points) The graph of $f'(x)$ is given below. Suppose $f(0) = 0$. Sketch the graph of $f(x)$ on the same coordinate system. Notice that f' is defined only on the interval $[-2, 5]$.



14. (10 points) Find a function $G(x)$ that satisfies $G'(x) = 3x^2 - 7x$ and $G(4) = 9$.

15. (40 points) Compute each of the following derivatives.

(a) $\frac{d}{dx} \sqrt{x^4 + 1}$

(b) $\frac{d}{dx} [\ln(2x + 1)]^3$

(c) $\frac{d}{dx} x^2 e^{2x+1}$

(d) $\frac{d}{dx} \frac{2x^2 - 3}{2x + 3}$

16. (20 points) Compute the following integrals.

(a) $\int_0^2 2xe^{-x^2} dx$

(b) $\int_0^5 (2x - 1)\sqrt{x^2 - x + 5} dx$

17. (10 points) Find the area of the region bounded by $y = x^{3/2}$, the x -axis, and the lines $x = 0$ and $x = 4$.

18. (15 points) Find the area A of the region caught between the graphs of the functions

$$f(x) = -x^2 + 4x \text{ and } g(x) = -2x + 5.$$