

December 14, 2016

Name _____

The problems count as marked. The total number of points available is xxx. Throughout this test, **show your work**. Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (48 points) Find the following antiderivatives.

(a) $\int 2x - 5 \, dx$

(b) $\int 9x^2 - 4x - 2/x \, dx$

(c) $\int \frac{3x^3 + 2x^2 - x}{x} \, dx$

(d) $\int \frac{2x + 3}{x^2 + 3x - 3} \, dx$

(e) $\int 6x^5(x^6 + 3)^7 \, dx$

(f) $\int x^2 e^{x^3} \, dx$

(g) $\int \frac{2x + 3}{x - 1} \, dx$. Try substitution with $u = x - 1$.

(h) $\int (x + 2)^4(x - 2) \, dx$

2. (10 points) Suppose you're given a function f to differentiate. You do so and you get $f'(x) = (x + 3)(x + 1)(x^2)(x - 4)$. You conclude that f has four critical points, $x = -3, -1, 0$ and 4 . Classify each of these as a) the location of a relative maximum, b) the location of a relative minimum, or c) an imposter.
3. (10 points) Consider the function $g(x) = \ln((x - 3)(x^2 + 5x + 4))$. Notice that $g(0) = \ln(-3 \cdot 4)$ is not defined because x must be positive in order to take \ln of it. Find the domain of $g(x)$ and put your answer in interval notation.
4. (12 points) Let $H(x) = 2x - \ln(4x^2 + 12x + 10)$. Find all the critical points.
5. (20 points) Consider the function $f(x) = (2x - 4)e^{x^2}$.
- (a) Use the product rule to find $f'(x)$.
- (b) List the critical points of f .
- (c) Construct the sign chart for $f'(x)$.
- (d) Write in interval notation the interval(s) over which f is increasing.

6. (36 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. Find the critical points for each function and the intervals over which the function is increasing. You must show your work.

(a) Let $F(x) = (2x + 8)(4x - 6)$

(b) $G(x) = \frac{x^2 - 3x + 15/2}{2x - 1}$

(c) $K(x) = (x^2 - 4)^{18}$

7. (10 points) The line tangent to the graph of $g(x)$ at the point $(4, 6)$ has a y -intercept of 9. What is $g'(4)$?

8. (10 points) Find all the points (x, y) on the graph of $h(x) = 2x^2 - 4x$ where the tangent line has a slope equal to 5.

9. (15 points) Find a number b such that $\int_b^{2b} x^3 dx = 60$.

You must show your work to get any credit on this problem. Guessing b is not enough.

10. (15 points) The region R is bounded by the vertical lines $x = 1$ and $x = e^2$ and by the graph of $f(x) = 1 + \frac{1}{x}$ and the x -axis. Find the area of R .

11. (30 points) Let $A = (6, 6)$, $B = (1, 4)$, $C = (1, 0)$ and $D = (6, 0)$ be the vertices of a quadrilateral in the plane.

(a) Sketch the figure and use geometry to find the area of $ABCD$

(b) Find an equation for the linear function (the line) that goes through the points A and B . Give this function the name f .

(c) Use calculus to find the area of the region R defined as follows:

$$R = \{(x, y) : 1 \leq x \leq 6, 0 \leq y \leq f(x)\}$$

12. (40 points) In fewer than 200 words, say what you think this course was about?