Math 1120 **Calculus** **Final Exam**

May 7, 2003 Your name ______________________________

As usual, **show all your work**. If you used a calculator, explain in detail how you used it to solve the problem. There are 237 points available on this test.

1. (42 points) Find the following antiderivatives.

   (a) $\int 2x - 3dx$

   (b) $\int 6x^2 - 4x - 1dx$

   (c) $\int \frac{x^3 + 2x - 1}{x}dx$

   (d) $\int \frac{4x + 1}{2x^2 + x - 3}dx$

   (e) $\int 5x^4(x^5 + 3)^7dx$

   (f) $\int 3x^2e^{x^3}dx$

2. (30 points) Note that $g(x) = (x-1)(x-3)$ has two zeros in the interval $[0, 4]$.

   (a) Find the area of the 'triangular' region bounded by (i) the x-axis, (ii) the line $x = 4$, and (iii) the graph of $g(x)$.

   (b) Compute $\int_0^4 g(x)\,dx$.

   (c) Find the area of the region caught between the graph of $g(x)$ and the x-axis over the interval from $x = 0$ to $x = 4$. Explain why this is different from the number found in part b.

3. (20 points) Find a function $G(x)$ whose derivative is $3x^2 - 7x + 3$ and for which $G(2) = -3$.

4. (40 points) Let $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x-1}$, and $h(x) = 2x - 3$. Find each of the functions.

   (a) $\frac{d}{dx}(f \circ g(x))$

   (b) $h'(g'(x))$

   (c) $\frac{d}{dx}(h \circ h(x))$

   (d) $\frac{d}{dx}(h(x) \cdot (g(x))^2)$

   (e) $\frac{d}{dx}(h(x) \div g(x))$

5. (20 points) Let $g(x)$ be defined as follows: Let

$$g(x) = \begin{cases} e^{2x} & \text{if } x \leq 1 \\ \ln(x-1) & \text{if } x > 1 \end{cases}$$

(a) Compute the derivative of $g(x)$.

(b) What is the slope of the line tangent to the graph of $g(x)$ at the point $(0, 1)$.

(c) What is the slope of the line tangent to the graph of $g(x)$ at the point $(3, \ln(2))$.

(d) Find an equation for the line tangent to the graph of $g(x)$ at the point $(3, \ln(2))$.

6. (40 points) Suppose $u(x)$ is a function whose derivative is

$$u'(x) = (x^2 - 4)(x - 1)^2(x + 3)(x + 5).$$

Recall that a major theorem tells you the intervals over which $u(x)$ is increasing based on $u'(x)$.

(a) Find the critical points of $u(x)$.

(b) Use the Test Interval Technique to find the intervals over which $u(x)$ is increasing.

7. (25 points) Consider the function $h(x) = \sqrt{2x^3 - 3x^2 - 36x + 500}$ defined over the interval $[-5, 5]$.

   (a) Find $h'(x)$.

   (b) Find the critical points of $h(x)$.

   (c) Find the absolute maximum and absolute minimum of the $h(x)$ over its domain.

8. (20 points) Compute the following definite integrals.

   (a) $\displaystyle\int_0^2 2xe^{-x^2}\,dx$

   (b) $\displaystyle\int_0^5 (2x-1)\sqrt{x^2 - x + 5}\,dx$