December 11, 2007 **Name** ________________________________

The total number of points available is 238. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = xe^{2x}$.

   (a) Find a value of x at which the line tangent to the graph of f is horizontal.

   (b) Find a value of x at which the line tangent to the graph of f has slope $2e$.

   (c) Find an equation of the tangent line referred to in part (b).

2. (30 points) Suppose $u(x)$ is a function whose derivative is

$$u'(x) = (x^2 - 9)(x - 1)^2(x - 7)(3x + 11).$$

   What this says is that u has already been differentiated and the function given is $u'(x)$ Recall that an important theorem tells you the intervals over which $u(x)$ is increasing based on $u'(x)$.

   (a) Find the critical points of $u(x)$.

   (b) Use the Test Interval Technique to find the intervals over which $u(x)$ is increasing.

3. (12 points) Given $f''(x) = 2x - 6$ and $f'(-2) = 6$ and $f(-2) = 0$. Find $f'(x)$ and $f(x)$.

4. (15 points) Compound Interest.

   (a) Consider the equation $1000(1+0.02)^{4t} = 5000$. Find the value of t and interpret your answer in the language of compound interest.

   (b) Consider the equation $P(1+0.03)^{4\cdot 10} = 5000$. Solve for P and interpret your answer in the language of compound interest.

   (c) Consider the equation $1000e^{10r} = 5000$. Solve for r and interpret your answer in the language of compound interest.

5. (12 points) Let $f(x) = \dfrac{6}{x} - 2e^x$.

   (a) Find an antiderivative of $f(x)$.

   (b) Compute $\displaystyle\int_1^e f(x)dx$.

6. (42 points) Find the following antiderivatives.

   (a) $\displaystyle\int 4x - 5\ dx$

   (b) $\displaystyle\int 9x^2 - 4x - 1/x\ dx$

   (c) $\displaystyle\int \frac{x^3 + 2x^2 - x}{x}\ dx$

   (d) $\displaystyle\int \frac{2x+3}{x^2+3x-3}\ dx$

   (e) $\displaystyle\int 6x^5(x^6+3)^7\ dx$

   (f) $\displaystyle\int x^2 e^{x^3}\ dx$

7. (40 points) This question is about building more complicated functions from simpler ones. Let $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = x + 1$, $k(x) = 1/x$ and $l(x) = x - 2$. For each function given below, show how it is possible to combine some of the simpler functions above to obtain the given one. For example, if $U(x) = \sqrt{x^2 - 2}$ was given, you could write $U(x) = g \circ l \circ f(x)$, and if $V(x) = ((x+1)/x)^2$, you could write $V(x) = f \circ (h \cdot k)(x)$.

   (a) $H(x) = \left(\frac{1}{x-2}\right)^2 + 1$

(b) $G(x) = \left(\frac{1}{x-2} + 1\right)^2$

(c) $L(x) = \frac{x}{x-2} - 2$

(d) $K(x) = \frac{1}{(x+1)^2-2}$

(e) $N(x) = (\sqrt{x^2+x+1} - 2)^2$

8. (15 points) Find the intervals over which $f(x) = x^2 e^{2x}$ is increasing.

9. (12 points) Is there a value of b for which $\int_b^{2b} x^4 + x^2\, dx = 128/15$? If so, find it.

10. (25 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the temperature of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $70°F$, A and k are constants, and t is expressed in minutes.

(a) What is the value of A?

(b) Suppose that after exactly 20 minutes, the temperature of the coffee is $186.6°F$. What is the value of k?

(c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

(d) Find the rate at which the object is cooling after $t = 20$ minutes.

11. (20 points) Amber Airlines runs chartered flights to Costa Rica. They expect 300 passengers and they charge each passenger \$200. However if more than 300 persons sign up for the flight, they agree to charge \$0.25 less per ticket for each extra person. For example, if 302 passengers sign up, the airline charges each of the 302 passengers \$199.50.

    (a) Find the revenue function $R(x)$ in terms of the number of new passengers x. In other words, let $x + 300$ represent the number of passengers, where $x > 0$.

    (b) How many passengers result in the maximum revenue?

    (c) What is that maximum revenue?