May 7, 2009 **Name** ________________________________

The total number of points available is 300. Throughout the free response part of this test, **show your work.** Throughout this test, the symbols DNE will mean 'does not exist' and $NOTA$ means 'none of the above'. In each of the first problems, circle the option that is closest to the correct answer. Each of the first 20 problems is worth 10 points.

1. Let $f(x) = x^3 - 2x + 4$. What is $f'(1)$?

   (A) 0 **(B)** 1 **(C)** 4 **(D)** 5 **(E)** $NOTA$

2. What is the y-intercept of the line tangent to the graph of $y = 3x^3 - 2x + 4$ at the point $(1, f(1))$?

   (A) 0 **(B)** -1 **(C)** -2 **(D)** -5 **(E)** $NOTA$

3. Consider the function $f(x) = xe^{2x}$. What is the slope of line tangent to the graph of f at the point $(\ln(3), 9\ln(3))$?

   (A) $3 + 6\ln(3)$ **(B)** $6\ln(3)$ **(C)** $9 + 18\ln(3)$

   (D) $9(1 + \ln(3))$ **(E)** $18(1 + \ln(3))$

4. Suppose the line $3x+2y = 7$ is tangent to the graph of $h(x)$ at the point $(1, 2)$. What is $h'(1)$?

   (A) $-3/2$ **(B)** $-2/3$ **(C)** 0 **(D)** $3/2$ **(E)** 7

5. What is $\lim_{x\to\infty} \dfrac{(x-2)(2x-3)}{(3x+2)(4x-1)}$?

   (A) 0 **(B)** 1/3 **(C)** 1/2 **(D)** 1/6 **(E)** DNE

6. What is the value of $|2\pi - 7| + |1 - \pi| + \pi$?

   (A) 0 **(B)** $4\pi - 6$ **(C)** 6 **(D)** $6 - 2\pi$ **(E)** $8 + 4\pi$

7. What is $\lim_{x\to 2} \dfrac{x^3 - 8}{x^2 - 4}$?

   (A) -3 **(B)** -2 **(C)** 2 **(D)** 3 **(E)** DNE

8. What is $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x^2 - 16}$?

   (A) 1/4 **(B)** 1/16 **(C)** 1/32 **(D)** 1/64 **(E)** DNE

9. Let $F(x)$ be an antiderivative of $3x^2 - 2x$. What is the growth of $F(x)$ over the interval $[1, 5]$?

   (A) 18 **(B)** 27 **(C)** 36 **(D)** 100

   (E) The answer depends on which antiderivative is selected.

10. Recall that one definition of $f'(x)$ is $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Use this idea to find $\lim_{h\to 0} \frac{(2+h)^6-64}{h}$.

    (A) 32 **(B)** 64 **(C)** 128 **(D)** 192 **(E)** DNE

11. Let $H(x) = \ln(4x^2 + 12x + 10) - 2x$. One critical point of $H(x)$ is $x = -1$. Find another critical point.

    (A) $x = 0$ **(B)** $x = 2$ **(C)** $x = e$ **(D)** $x = 4$

    (E) There are no other critical points.

12. Let $g(x) = 2x^3 - 7x^2 + 4x - 10$. Over which one of the following intervals is g is increasing?

    (A) $[-2, 1]$ **(B)** $[-1, 2]$ **(C)** $[0, 3]$ **(D)** $[1, 4]$ **(E)** $[2, 5]$

13. Let $k(x) = 2x^4 - 14x^3 + 30x^2 + 10x$. Over which of the following intervals is k is concave downwards?

    (A) $(0, 1)$ **(B)** $(1, 2)$ **(C)** $(2, 3)$ **(D)** $(3, 4)$ **(E)** $(4, 5)$

14. What is the value of $\int_2^4 \frac{d(2x-5)^4}{dx}\ dx$

    (A) 20 **(B)** 40 **(C)** 60 **(D)** 80 **(E)** 100

15. What is the area of the region R bounded above by $y = 2x - 3$, below by $y = x - 7$, on the left by $x = 2$ and on the right by $x = 4$?

    (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

16. Find a value of b for which $\displaystyle\int_b^{2b} \frac{1}{x} + 1\, dx = \ln(2) + 6.$

    (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

17. The absolute maximum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 4$ on the interval $-2 \leq x \leq 3$ is

    (A) -10 **(B)** 0 **(C)** 9 **(D)** 12 **(E)** 13

18. Two of the zeros of the polynomial $p(x) = (x-1)^3(x+2)^2 - 4(x-1)^2(x+2)$ are $x = 1$ and $x = -2$. There are two others. What is their sum?

    (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

19. Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after t weeks is given by

$$F(t) = 120 - 40e^{-.4t}$$

    Which of the following is closest to the increase in the number of words per minute Rachel can type during the second week of the course?

    (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

20. Given that the graph of f passes through the point $(1, 5)$ and that the slope of its tangent line at $(x, f(x))$ is $2x + 1$, what is $f(4)$?

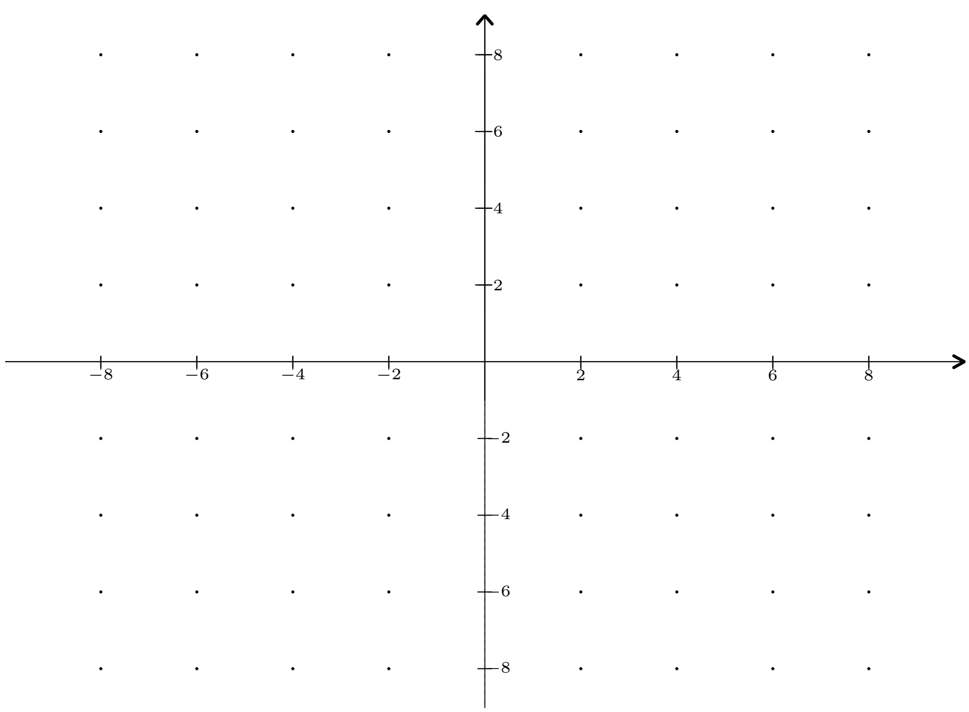
    (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 28

21. (30 points) Let $h(x) = \frac{x(2x+11)(2x+7)}{(x-1)^2(3x-12)}$.

(a) Find the asymptotes and the zeros of h.

(b) Build the sign chart for $h(x)$.

(c) Sketch the graph of $h(x)$ USING the information in (a) and (b).



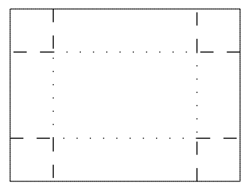
22. (20 points) Let $H(x) = \sqrt{(2x+1)^{10}+3}$.

    (a) Find three functions f, g and h satisfying $f(g(h(x))) = f \circ g \circ h(x) = H(x)$.

    (b) Compute the derivative of each of the three component functions f, g, h.

    (c) Apply the chain rule twice to find $H'(x)$.

23. (20 points) For the problem below, let F be the number of letters in your first (= given) name, and let L be the number of letters in your last (= family) name. The point is to customize the problems for you. Four congruent $x \times x$ squares from the corners of a cardboard rectangle that measures $2F \times 2L$. The sides are then folded upward to form a topless box. Find the volume V as a function of x. What is the logical domain? Compute $V(0), V(1), V(2)$, and $V(3)$. Find $V'(x)$ and use this to determine the critical points of V. Find the absolute maximum value of V and the value of x where it occurs.



24. (30 points) The quadrilateral T with vertices $A = (0,0)$, $B = (0,6)$, $C = (8,10)$ and $D = (8,0)$ is a trapezoid since the two sides AB and CD are both vertical. It is not hard to see that the area of T is 64 square units.

    (a) Find an equation for the line passing through the points B and C. Let $f(x)$ be the function whose graph is this line.

    (b) Use calculus, showing all you work, to verify that the area of the region T bounded above by the graph of f, below by the x-axis, and on the sides by $x = 0$ and $x = 8$ is 64.