

December 14, 2009

Name \_\_\_\_\_

The total number of points available is 282. Throughout this test, **show your work**. Throughout this test, the symbols *DNE* will mean 'does not exist'. In each of the first problems, circle the option that is closest to the correct answer. Each of the first 19 problems is worth 10 points.

- Let  $f(x) = x^4 - 5x + 4$ . What is  $f'(1)$ ?  
(A)  $-1$     (B)  $1$     (C)  $3$     (D)  $5$     (E)  $7$
- Which of the following intervals contains the number  $|2\pi - 3\sqrt{5}| + |2\pi + \sqrt{5} - 8|$ ?  
(A)  $[0, 0.2)$     (B)  $[0.2, 0.4)$     (C)  $[0.4, 0.6)$     (D)  $[0.6, 0.8)$     (E)  $[0.8, 1]$
- What is the slope of a line perpendicular to the line  $3x + 2y = 7$ ?  
(A)  $2/3$     (B)  $3/2$     (C)  $-2/3$     (D)  $-3/2$     (E) None of the above
- Which of the following belongs to the domain of  $f(x) = \ln((x^2 + x - 2)(x^2 + 2x - 15))$ ?  
(A)  $-4$     (B)  $-2$     (C)  $-1$     (D)  $1$     (E)  $2$
- Suppose the line  $3x + 2y = 7$  is tangent to the graph of  $h(x)$  at the point  $(1, 2)$ . What is  $h'(1)$ ?  
(A)  $-3/2$     (B)  $-2/3$     (C)  $0$     (D)  $3/2$     (E)  $7$
- What is  $\lim_{x \rightarrow \infty} \frac{(6x - 2)(2x - 3)}{(3x + 2)(4x - 1)(x - 1)}$ ?  
(A)  $0$     (B)  $1/3$     (C)  $1/2$     (D)  $1/6$     (E)  $1$

7. What is  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 4}$ ?
- (A)  $-3$     (B)  $-2$     (C)  $2$     (D)  $3$     (E)  $DNE$
8. Let  $F(x)$  be an antiderivative of  $3x^2 - 2x$ . What is the growth of  $F(x)$  over the interval  $[1, 5]$ ?
- (A)  $18$     (B)  $27$     (C)  $36$     (D)  $100$
- (E) The answer depends on which antiderivative is selected.
9. Let  $H(x) = \ln(4x^2 + 12x + 10) - 2x$ . One critical point of  $H(x)$  is  $x = -1$ . Find another critical point.
- (A)  $x = 0$     (B)  $x = 2$     (C)  $x = e$     (D)  $x = 4$
- (E) There are no other critical points.
10. Let  $g'(x) = (x - 4)(x - 2)(x + 3)$ . Over which one of the following intervals is  $g$  increasing?
- (A)  $[-6, -3]$     (B)  $[-2, 1]$     (C)  $[0, 3]$     (D)  $[1, 4]$     (E)  $[3, 6]$

11. Which of the following is closest to the time required for a 12% investment to triple in value?  
(A) 5 years    (B) 7 years    (C) 9 years    (D) 11 years    (E) 13 years
12. The half-life of a radioactive material is 100 years. How long does it take the material to lose one-third of its radioactivity?  
(A) 32 years    (B) 40 years    (C) 50 years    (D) 58 years    (E) 62 years
13. What is the value of  $\int_2^4 \frac{d(2x-5)^2}{dx} dx$   
(A) 2    (B) 4    (C) 6    (D) 8    (E) 10
14. What is the area of the region  $R$  bounded above by  $y = 2x - 3$ , below by  $y = x - 7$ , on the left by  $x = 2$  and on the right by  $x = 6$ ?  
(A) 20    (B) 24    (C) 28    (D) 32    (E) 36
15. Find a value of  $b$  for which  $\int_b^{2b} x^2 dx = 63$ .  
(A) 2    (B) 3    (C) 4    (D) 5    (E) 7

16. The absolute maximum value of the function  $f(x) = 2x^3 - 9x^2 + 12x + 14$  on the interval  $-2 \leq x \leq 3$  is

- (A) 0    (B) 10    (C) 19    (D) 22    (E) 23

17. Two of the zeros of the polynomial  $p(x) = (x - 1)^3(x + 2)^2 - 4(x - 1)^2(x + 2)$  are  $x = 1$  and  $x = -2$ . There are two others. What is their sum?

- (A) -2    (B) -1    (C) 0    (D) 1    (E) 2

18. Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after  $t$  weeks is given by

$$F(t) = 160 - 40e^{-.4t}$$

During which week does Rachel attain a speed of at least 155 words per minute.

- (A) 3    (B) 4    (C) 5    (D) 6    (E) 7

19. Consider the function  $f(x) = xe^{2x}$ . What is the slope of line tangent to the graph of  $f$  at the point  $(\ln(3), 9\ln(3))$ ?

- (A)  $3 + 6\ln(3)$     (B)  $6\ln(3)$     (C)  $9 + 18\ln(3)$   
(D)  $9(1 + \ln(3))$     (E)  $18(1 + \ln(3))$

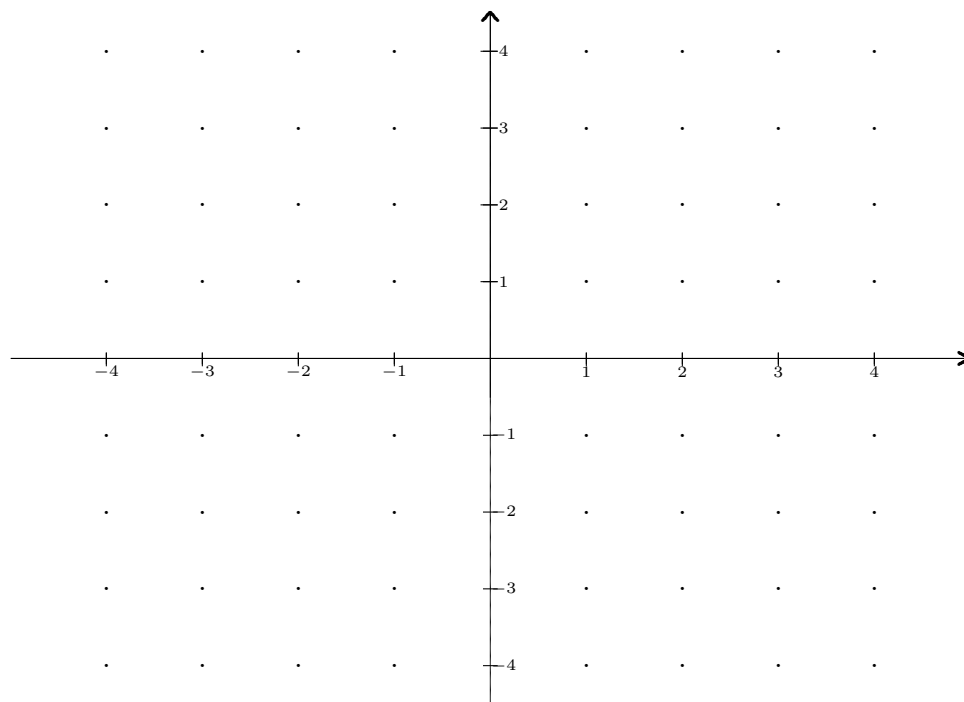
20. (30 points) Let  $h(x) = \frac{x(2x-3)(2x+7)}{(x-1)^2(3x+12)}$ .

(a) Find the zeros of  $h$ .

(b) Find the asymptotes of  $h$ .

(c) Build the sign chart for  $h(x)$ .

(d) Sketch the graph of  $h(x)$  USING the information in (a) and (b).



21. (20 points) Let  $H(x) = \sqrt{(3x + 1)^9 + 2}$ .

(a) Find three functions  $f, g$  and  $h$  satisfying  $f(g(h(x))) = f \circ g \circ h(x) = H(x)$ .

(b) Compute the derivative of each of the three component functions  $f, g, h$ .

(c) Apply the chain rule twice to find  $H'(x)$ .

22. (42 points) Find the following antiderivatives and definite integrals.

(a)  $\int 3x - 5 \, dx$

(b)  $\int_1^5 3x^2 - 2x + 7 \, dx$

(c)  $\int_0^1 \frac{d}{dx} e^{x^2} \, dx$

(d)  $\int_e^3 \frac{1}{x} \, dx$

(e)  $\int_0^5 \frac{2x}{x^2 + 1} \, dx$

(f)  $\int_0^1 2x(x^2 + 1)^4 \, dx$