

May 12, 2010 Name \_\_\_\_\_

The total number of points available is 260. Throughout this test, the symbols *DNE* will mean 'does not exist'. In each problem, circle the option that is closest to the correct answer.

- Let  $f(x) = x^5 - 5x + 4$ . What is  $f'(1)$ ?  
(A) 0 (B) 1 (C) 3 (D) 5 (E) 7
- What is the  $y$ -intercept of the line tangent to the graph of  $f(x) = 2x^2 - 5x$  at the point  $(1, -3)$ ?  
(A)  $-2$  (B)  $-1$  (C)  $0$  (D)  $1$  (E)  $2$
- How many solutions does the equation  $|x^2 - 8| = 1$  have?  
(A)  $0$  (B)  $1$  (C)  $2$  (D)  $3$  (E)  $4$
- What is the slope of a line perpendicular to the line  $5x + 2y = 7$ ?  
(A)  $2/5$  (B)  $5/2$  (C)  $-2/5$  (D)  $-5/2$  (E) None of the above
- Which of the following belongs to the domain of  $f(x) = \ln((x^2 + x - 2)(x^2 + 2x - 15))$ ?  
(A)  $-4$  (B)  $-2$  (C)  $-1$  (D)  $1$  (E)  $2$
- Suppose the line  $3x - 2y = 7$  is tangent to the graph of  $h(x)$  at the point  $(1, 2)$ . What is  $h'(1)$ ?  
(A)  $-3/2$  (B)  $-2/3$  (C)  $0$  (D)  $3/2$  (E)  $7$

7. What is  $\lim_{x \rightarrow \infty} \frac{(6x - 2)(2x - 3)}{(3x + 2)(4x - 1)(x - 1)}$ ?
- (A) 0    (B)  $1/3$     (C)  $1/2$     (D)  $1/6$     (E) *DNE*
8. What is  $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^3 + 8}$ ?
- (A)  $-1/3$     (B)  $-1/2$     (C)  $1/2$     (D)  $1/3$     (E) *DNE*
9. Let  $F(x)$  be an antiderivative of  $x^2 - 2x$ . What is the growth of  $F(x)$  over the interval  $[0, 6]$ ?
- (A) 18    (B) 27    (C) 36    (D) 100
- (E) The answer depends on which antiderivative is selected.
10. Let  $H(x) = \ln(12x + 10) - 2x$ . Find a critical point.
- (A)  $x = -1/3$     (B)  $x = 0$     (C)  $x = 1/3$
- (D)  $x = 1$     (E)  $x = 4/3$
11. Let  $g'(x) = (x - 6)(x - 2)(x + 3)$ . Over which one of the following intervals is  $g$  is increasing?
- (A)  $[-4, -2]$     (B)  $[-2, 0]$     (C)  $[0, 3]$     (D)  $[3, 4]$     (E)  $[5, 7]$

12. Which of the following is closest to the time required for a 10% investment to triple in value if compounding is continuous?  
(A) 7 years    (B) 9 years    (C) 11 years    (D) 12 years    (E) 13 years
13. Which of the following is closest to the time required for a 10% investment to triple in value if compounding is quarterly?  
(A) 7 years    (B) 9 years    (C) 11 years    (D) 12 years    (E) 13 years
14. The half-life of a radioactive material is 100 years. How long does it take the material to lose two-thirds of its radioactivity?  
(A) 132 years    (B) 140 years    (C) 150 years  
(D) 158 years    (E) 162 years
15. What is the value of  $\int_2^4 \frac{d}{dx} (3x - 5)^2 dx$ ?  
(A) 24    (B) 44    (C) 46    (D) 48    (E) 60
16. What is the area of the region  $R$  bounded above by  $y = 2x - 3$ , below by  $y = x - 7$ , on the left by  $x = 2$  and on the right by  $x = 6$ ?  
(A) 20    (B) 24    (C) 28    (D) 32    (E) 36
17. Find a value of  $b$  for which  $\int_b^{2b} x^2 dx = 56/3$ .  
(A) 2    (B) 3    (C) 4    (D) 5    (E) 7

18. What is the absolute maximum value of the function  $f(x) = x^3 - 9x^2 + 24x$  on the interval  $1 \leq x \leq 5$ ?

- (A)  $-10$    (B)  $0$    (C)  $9$    (D)  $16$    (E)  $20$

19. Two of the zeros of the polynomial  $p(x) = (x - 1)^3(x + 2)^2 - 4(x - 1)^2(x + 2)$  are  $x = 1$  and  $x = -2$ . There are two others. What is the sum of the two others?

- (A)  $-2$    (B)  $-1$    (C)  $0$    (D)  $1$    (E)  $2$

20. Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after  $t$  weeks is given by

$$F(t) = 160 - 40e^{-.4t}.$$

During which week does Rachel attain a speed of at least 135 words per minute?

- (A) week 1   (B) week 2   (C) week 3   (D) week 4   (E) week 5

21. Consider the function  $f(x) = xe^{2x}$ . What is the slope of line tangent to the graph of  $f$  at the point  $(\ln(2), 4\ln(2))$ ?

- (A)  $4 + 2\ln(2)$    (B)  $4\ln(2)$    (C)  $4 + 4\ln(2)$   
(D)  $8\ln(2)$    (E)  $4(1 + 2\ln(2))$

22. If  $f(x) = x^3(x^2 + 2x)$ , then  $f'(x) =$

- (A)  $3x^2(x^2 + 2x) + x^3(2x + 2)$     (B)  $x^3(x^2 + 2x)$     (C)  $3x^2(x^2 + 2x)$   
(D)  $3x^2(2x + 2)$     (E)  $3x^2(x^2 + 2x) + x^3(3x)$

23. If  $g(x) = 3\sqrt{x} + \frac{1}{x^2}$ , then  $g'(x) =$

- (A)  $-3x^{-2} + \frac{1}{2x}$     (B)  $-3x^{-2} + 2x$     (C)  $\frac{3}{2}x^{-1/2} + \frac{1}{2x}$   
(D)  $3 + \frac{1}{2x}$     (E)  $\frac{3}{2}x^{-1/2} - 2x^{-3}$

24. If  $f(x) = (2x^2 + 1)^4$ , then  $f'(x) =$

- (A)  $4(2x^2 + 1)^3$     (B)  $4(2x^2 + 1)^3 \cdot 4x$     (C)  $4(4x)^3$   
(D)  $(4x)^4$     (E)  $4(4x)^3 \cdot 4x$

25. If  $f(t) = e^{t-1} + \ln(t)$ , then  $f'(1) =$

- (A) 0    (B) 1    (C) 2    (D) 3    (E)  $e^2$

26. If  $f(x) = 2e^{2x^2+1}$ , then  $f'(x) =$

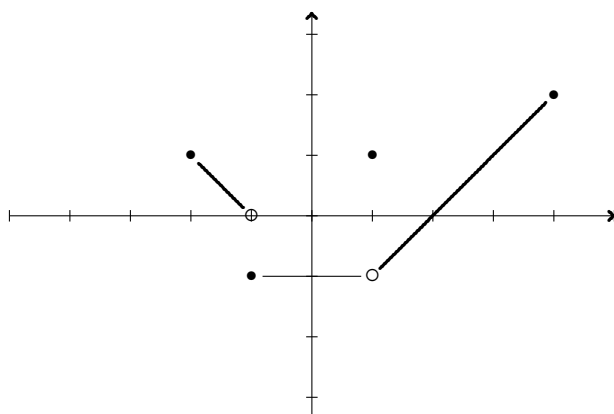
- (A)  $2e^{4x}$     (B)  $e^{2x^2+1} \cdot 4x$     (C)  $e^{4x}$     (D)  $2e^{2x^2+1} \cdot 4x$     (E)  $2e^{2x^2+1} + 2e^{2x^2+1} \cdot 4x$

27.  $\int(2x^3 + x + 4) dx =$

- (A)  $\frac{1}{4}x^4 + \frac{1}{2}x^2 + 4x + C$     (B)  $\frac{1}{2}x^4 + \frac{1}{2}x^2 + 4 + C$     (C)  $\frac{1}{2}(2x^3 + x + 4)^2 + C$   
(D)  $\frac{1}{2}x^4 + \frac{1}{2}x^2 + C$     (E)  $\frac{1}{2}x^4 + \frac{1}{2}x^2 + 4x + C$

28.  $\int_1^4 (2x + 1) dx =$

- (A) 0    (B) 6    (C) 15    (D) 18    (E) 20

Consider the graph of the function  $f$ :

29. Based on the graph,  $\lim_{x \rightarrow 1} f(x) =$

- (A) -1    (B) 0    (C) 1    (D) 2    (E) DNE

30. Again referring to the graph above, what is  $\lim_{x \rightarrow -1} f(x) =$

- (A) -1    (B) 0    (C) 1    (D) 2    (E) DNE

31.  $\lim_{x \rightarrow 0} \frac{x}{x^2 + 2x} =$

- (A) 0    (B) 1    (C) 1/2    (D) 1/3    (E) DNE

32. Let  $f(x) = \frac{x}{2x + 1}$ . What is the slope of the tangent line to the graph of  $f$  at  $x = 2$ ?

- (A) -1/2    (B) -1/5    (C) 0    (D) 1/25    (E) 1/5

33. Let  $f(x) = x^3 - 12x + 1$ . Which of the following is correct?

(A)  $f$  is increasing on  $(-\infty, \infty)$ .

(B)  $f$  is decreasing on  $(-\infty, \infty)$ .

(C)  $f$  is increasing on  $(-2, 2)$ .

(D)  $f$  is decreasing on  $(-2, 2)$ .

(E)  $f$  is increasing on  $(-\infty, 2)$  and decreasing on  $(2, \infty)$ .

34. Let  $f(x) = x^3 - 3x^2 + 2x + 50$ . Then  $f$  has a point of inflection at  $x =$

(A) 0    (B) 1    (C) 2    (D) 3    (E) 4

solB. Since  $f'(x) = 3x^2 - 6x + 2$ , it follows that  $f''(x) = 6x - 6$ , so there is a change in sign at  $x = 1$ .

35. Let  $f(x) = \ln(x) + x$ . Which of the following is the equation of the tangent line to the graph of  $f$  at  $x = 1$ ?

(A)  $y - 1 = \left(\frac{1}{x} + 1\right)(x - 1)$

(B)  $y - 2 = x - 1$

(C)  $y - 1 = 2(x - 2)$

(D)  $y - 2 = 2(x - 1)$

(E)  $y - 1 = 2(x - 1)$

36. Wacky Widgets, Inc. earns a daily profit of  $P(x) = -10x^2 + 1760x - 50,000$  dollars when it produces  $x$  tons of widgets. Which of the following gives the marginal profit at a production level of 50 tons.
- (A)  $-50,000$     (B) 0    (C) 760    (D) 1000    (E) 13,000
37. For a certain function  $g$ , it is known that  $g'(x) = e^x + 2x$  and that  $g(0) = 5$ . Which of the following is closest to  $g(2)$ ?
- (A) 7.39    (B) 9.39    (C) 11.39    (D) 13.39    (E) 15.39
38. What is  $\lim_{x \rightarrow \infty} \frac{1 + 2e^x}{e^x}$ ?
- (A) 0    (B) 1    (C) 2    (D) 3    (E)  $\infty$
39. How many asymptotes, both horizontal and vertical, does  $r(x) = \frac{(x-2)(x-1)(x^2)}{x(x^2-1)}$  have?
- (A) 1    (B) 2    (C) 3    (D) 4    (E) 5
40. The derivative  $f'(x) = 3x - 2$ , and  $f(2) = 5$ . What is  $f(1)$ ?
- (A)  $1/2$     (B)  $3/2$     (C)  $5/2$     (D)  $7/2$     (E)  $9/2$
41. Let  $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2 & \text{if } x \geq 1. \end{cases}$
- What is  $\lim_{x \rightarrow 1} f(x)$ ?
- (A) 1    (B) 2    (C) 3    (D) 4    (E) DNE
42. Let  $f(x) = 2x^2 - x + 3$ . The minimum value of  $f$  on  $[0, 1]$  is
- (A) 1.275    (B) 2.350    (C) 2.875    (D) 3.125    (E) 4.075



43. Joe (who did not do well in his calculus course) now works long hours at Wacky Widgets. His supervisor has timed his work and has determined that, on a good day, Joe will have assembled a total of  $N(t) = -t^3 + 6t^2 + 15t$  widgets  $t$  hours after starting work. At what rate is Joe assembling widgets 3 hours after starting work (on a good day)?

- (A) 0    (B) 24    (C) 27    (D) 66    (E) 72

44. You just ordered a new seedling from a seed catalog. If the seedling is 2 inches tall when you receive it and it will be growing at a rate of  $2t + 1$  inches per month  $t$  months after you receive it, how tall will it be in 5 months?

- (A) 2    (B) 11    (C) 25    (D) 30    (E) 32

Consider the function  $f(x) = \ln[(x^2 - 9)(x^2 - 16)]$ . The next three problems all refer to  $f$ .

45. Recall that  $\ln(x)$  is defined precisely when  $x > 0$ . At which of the following points is  $f$  undefined?

- (A) 0.5    (B) 1.5    (C) 2.5    (D) 3.5    (E) 4.5

46. Which of the following is a critical point of  $f$ ?

- (A) -9    (B) -5    (C) 1    (D) 2    (E) 7

47. Which of the following is a critical point of  $f$ ?

- (A) -6    (B) 0    (C) 6    (D) 8    (E) 9

48. What is the slope of the line tangent to  $f(x) = xe^{2x}$  at the point  $(1, e^2)$ ?

- (A)  $e^2$     (B)  $2e^2$     (C)  $3e^2$     (D)  $4e^2$     (E)  $5e^2$

49. Find the growth of  $g(x) = \ln(e^2 + x)$  over the interval  $[2e^2, 5e^2]$ .

- (A)  $\ln 2$     (B)  $\ln 3$     (C)  $\ln 6$     (D) 2    (E) 3

50. What is the minimum value that  $f(x) = x^3 - 6x^2$  attains over the interval  $[-1, 5]$ ?

- (A) 0    (B) 4    (C) -25    (D) -32    (E) -64

51. What is the slope of the line tangent to  $y = \sqrt{e^x + 3}$  at the point  $(0, 2)$ ?
- (A)  $1/8$    (B)  $1/4$    (C)  $1/2$    (D)  $1$    (E)  $-1$

52. For which values of  $x$  is the line tangent to  $g(x) = \sqrt{x^2 + 1}$  horizontal?
- (A)  $0$    (B)  $1$    (C)  $-1$    (D)  $1/2$    (E) There is no such  $x$ .