May 12, 2010 **Name** ______________________________

The total number of points available is 260. Throughout this test, the symbols DNE will mean 'does not exist'. In each problem, circle the option that is closest to the correct answer.

1. Let $f(x) = x^5 - 5x + 4$. What is $f'(1)$?

   (A) 0 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 7

   Solution: A. $f'(x) = 5x^4 - 5$, so $f'(1) = 5 - 5 = 0$.

2. What is the y-intercept of the line tangent to the graph of $f(x) = 2x^2 - 5x$ at the point $(1, -3)$?

   (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

   Solution: A. $f'(1) = -1$, so the line is $y+3 = -1(x-1)$ which has y-intercept -2.

3. How many solutions does the equation $|x^2 - 8| = 1$ have?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

   Solution: E. There are four solutions, $x = \pm\sqrt{7}$ and $x = \pm 3$.

4. What is the slope of a line perpendicular to the line $5x + 2y = 7$?

   (A) $2/5$ **(B)** $5/2$ **(C)** $-2/5$ **(D)** $-5/2$ **(E)** None of the above

   Solution: A. The slope of the given line is $-5/2$ so the slope we seek is $2/5$.

5. Which of the following belongs to the domain of $f(x) = \ln((x^2 + x - 2)(x^2 + 2x - 15))$?

   (A) -4 **(B)** -2 **(C)** -1 **(D)** 1 **(E)** 2

   Solution: C. Build the sign chart for the function $g(x) = (x+2)(x-1)(x+5)(x-3)$, and notice that $g(-1)$ is a positive number. None of the other options are in the domain of $f(x)$.

6. Suppose the line $3x - 2y = 7$ is tangent to the graph of $h(x)$ at the point $(1, 2)$. What is $h'(1)$?

   (A) $-3/2$ **(B)** $-2/3$ **(C)** 0 **(D)** $3/2$ **(E)** 7

   Solution: D. The slope of the line is $m = 3/2$.

7. What is $\lim_{x\to\infty} \dfrac{(6x-2)(2x-3)}{(3x+2)(4x-1)(x-1)}$?

   (A) 0 **(B)** 1/3 **(C)** 1/2 **(D)** 1/6 **(E)** DNE

   Solution: A. Using the asymptote theorem, since the degree of the denominator is larger, the limit is 0.

8. What is $\lim_{x\to -2} \dfrac{x^2-4}{x^3+8}$?

   (A) $-1/3$ **(B)** $-1/2$ **(C)** 1/2 **(D)** 1/3 **(E)** DNE

   Solution: A. Factor both numerator and denominator to get $\lim_{x\to -2} \frac{x^2-4}{x^3+8} = \lim_{x\to -2} \frac{(x-2)(x+2)}{(x+2)(x^2-2x+4)} = -4/12 = -1/3$

9. Let $F(x)$ be an antiderivative of $x^2 - 2x$. What is the growth of $F(x)$ over the interval $[0, 6]$?

   (A) 18 **(B)** 27 **(C)** 36 **(D)** 100

   (E) The answer depends on which antiderivative is selected.

   Solution: C. One antiderivative is $F(x) = x^3/3 - x^2$ which grows from 0 to 36 on the given interval.

10. Let $H(x) = \ln(12x + 10) - 2x$. Find a critical point.

    (A) $x = -1/3$ **(B)** $x = 0$ **(C)** $x = 1/3$

    (D) $x = 1$ **(E)** $x = 4/3$

    Solution: A. We need to solve the equation $\frac{12}{12x+10} = 2$. This is equivalent to $24x + 8 = 0$ which has repeated roots, $x = -1/3$

11. Let $g'(x) = (x-6)(x-2)(x+3)$. Over which one of the following intervals is g is increasing?

    (A) $[-4, -2]$ **(B)** $[-2, 0]$ **(C)** $[0, 3]$ **(D)** $[3, 4]$ **(E)** $[5, 7]$

    Solution: B. $g'(x) > 0$ on $(-3, 2)$, so $[-2, 1]$ is one of the intervals over which g is increasing. But $g'(x) < 0$ at some points of each of the others.

12. Which of the following is closest to the time required for a 10% investment to triple in value if compounding is continuous?

    (A) 7 years **(B)** 9 years **(C)** 11 years **(D)** 12 years **(E)** 13 years

    Solution: C. The triple time for continuous compounding is 10.98 years.

13. Which of the following is closest to the time required for a 10% investment to triple in value if compounding is quarterly?

    (A) 7 years **(B)** 9 years **(C)** 11 years **(D)** 12 years **(E)** 13 years

    Solution: C. The triple time for quarterly compounding is 11.12 years.

14. The half-life of a radioactive material is 100 years. How long does it take the material to lose two-thirds of its radioactivity?

    (A) 132 years **(B)** 140 years **(C)** 150 years

    (D) 158 years **(E)** 162 years

    Solution: D. It takes 158.5 years to lose down to 1/3 of its radioactivity.

15. What is the value of $\int_2^4 \frac{d}{dx}\,(3x-5)^2$?

    (A) 24 **(B)** 44 **(C)** 46 **(D)** 48 **(E)** 60

    Solution: D. Its just $(3x-5)^2|_2^4 = 7^2 - (1)^2 = 49 - 1 = 48$.

16. What is the area of the region R bounded above by $y = 2x - 3$, below by $y = x - 7$, on the left by $x = 2$ and on the right by $x = 6$?

    (A) 20 **(B)** 24 **(C)** 28 **(D)** 32 **(E)** 36

    Solution: D. Let $f(x) = 2x-3-(x-7) = x+4$. Now $\int_2^4 x+4 = x^2/2+4x|_2^6 = 18 + 24 - (2 + 8) = 32$. Alternatively, you could do this by geometry because the region in question is the union of two trapezoids.

17. Find a value of b for which $\displaystyle\int_b^{2b} x^2\,dx = 56/3$.

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

    Solution: A. Solve $\frac{x^3}{3}|_b^{2b} = 7b^2/3 = 56/3$ for b to get $b^3 = 8$ and $b = 2$.

18. What is the absolute maximum value of the function $f(x) = x^3 - 9x^2 + 24x$ on the interval $1 \leq x \leq 5$?

    (A) -10 **(B)** 0 **(C)** 9 **(D)** 16 **(E)** 20

    Solution: E. Find $f'(x)$ first and then the critical points that are between 1 and 5. $f'(x) = 3x^2 - 18x + 24 = 3(x^2 - 6x + 8) = 3(x-2)(x-4)$, so there are two critical points and two endpoints to check: $f(2) = 20; f(4) = 16; f(1) = 16;$ and $f(5) = 20$, so the absolute maximum is $f(2) = f(5) = 20$.

19. Two of the zeros of the polynomial $p(x) = (x-1)^3(x+2)^2 - 4(x-1)^2(x+2)$ are $x = 1$ and $x = -2$. There are two others. What is the sum of the two others?

    (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

    Solution: B. Factor p to get $p(x) = (x-1)^2(x+2)[(x-1)(x+2) - 4]$. The sum of the two zeros of $(x-1)(x+2) - 4 = x^2 + x - 6 = (x+3)(x-2)$ is -1.

20. Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after t weeks is given by

    $$F(t) = 160 - 40e^{-.4t}.$$

    During which week does Rachel attain a speed of at least 135 words per minute?

    (A) week 1 **(B)** week 2 **(C)** week 3 **(D)** week 4 **(E)** week 5

    Solution: B. Solve $F(t) = 135 = 160 - 40e^{-0.4t}$, so $t = \ln(5/8) \div -0.4 \approx 1.17$. Alternatively, Rachel's speed at the end of 1 weeks is just below 135, so it is during the second week that her speed goes over 135.

21. Consider the function $f(x) = xe^{2x}$. What is the slope of line tangent to the graph of f at the point $(\ln(2), 4\ln(2))$?

    (A) $4 + 2\ln(2)$ **(B)** $4\ln(2)$ **(C)** $4 + 4\ln(2))$

    (D) $8\ln(2))$ **(E)** $4(1 + 2\ln(2))$

    Solution: E. Since $f'(x) = e^{2x} + 2xe^{2x}$ by the product rule, $f'(\ln(2)) = 4(1 + 2\ln(2))$.

22. If $f(x) = x^3(x^2 + 2x)$, then $f'(x) =$

    (A) $3x^2(x^2 + 2x) + x^3(2x + 2)$ **(B)** $x^3(x^2 + 2x)$ **(C)** $3x^2(x^2 + 2x)$

    (D) $3x^2(2x + 2)$ **(E)** $3x^2(x^2 + 2x) + x^3(3x)$

    Solution: A. By the product rule, $f'(x) = 3x^2(x^2 + 2x) + x^3(2x + 2)$.

23. If $g(x) = 3\sqrt{x} + \frac{1}{x^2}$, then $g'(x) =$

    (A) $-3x^{-2} + \dfrac{1}{2x}$ **(B)** $-3x^{-2} + 2x$ **(C)** $\dfrac{3}{2}x^{-1/2} + \dfrac{1}{2x}$

    (D) $3 + \dfrac{1}{2x}$ **(E)** $\dfrac{3}{2}x^{-1/2} - 2x^{-3}$

    Solution: E. By the power rule applied twice, $g'(x) = \frac{3}{2}x^{-1/2} - 2x^{-3}$.

24. If $f(x) = (2x^2 + 1)^4$, then $f'(x) =$

    (A) $4(2x^2 + 1)^3$ **(B)** $4(2x^2 + 1)^3 \cdot 4x$ **(C)** $4(4x)^3$

    (D) $(4x)^4$ **(E)** $4(4x)^3 \cdot 4x$

    Solution: B. By the chain rule, $f'(x) = 4(2x^2 + 1)^3 \cdot 4x$.

25. If $f(t) = e^{t-1} + \ln(t)$, then $f'(1) =$

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** e^2

    Solution: C. Since $f'(t) = e^{t-1} + 1/t$, it follows that $f'(1) = 2$.

26. If $f(x) = 2e^{2x^2+1}$, then $f'(x) =$

    (A) $2e^{4x}$ **(B)** $e^{2x^2+1} \cdot 4x$ **(C)** e^{4x} **(D)** $2e^{2x^2+1} \cdot 4x$ **(E)** $2e^{2x^2+1} + 2e^{2x^2+1} \cdot 4x$

    Solution: D. By the chain rule, $f'(x) = 2e^{2x^2+1} \cdot 4x$.

27. $\int (2x^3 + x + 4)\, dx =$

    (A) $\dfrac{1}{4}x^4 + \dfrac{1}{2}x^2 + 4x + C$ **(B)** $\dfrac{1}{2}x^4 + \dfrac{1}{2}x^2 + 4 + C$ **(C)** $\dfrac{1}{2}(2x^3 + x + 4)^2 + C$

    (D) $\dfrac{1}{2}x^4 + \dfrac{1}{2}x^2 + C$ **(E)** $\dfrac{1}{2}x^4 + \dfrac{1}{2}x^2 + 4x + C$

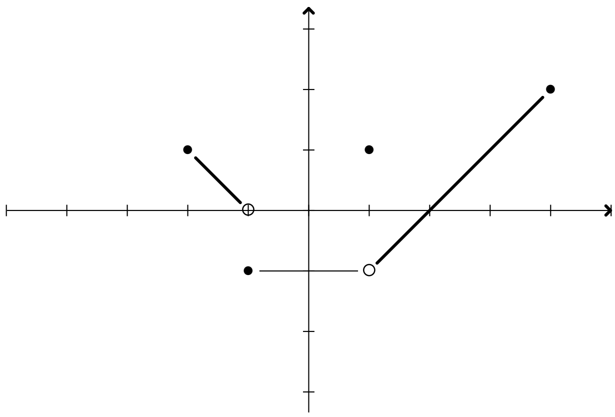
    Solution: E. Antidifferentiating term by term, we get E.

28. $\int_1^4 (2x+1)\,dx =$

    (A) 0 **(B)** 6 **(C)** 15 **(D)** 18 **(E)** 20

    Solution: D. Measure the growth of $x^2 + x$ over the interval 1 to 4 to get $4^2 + 4 - (1^2 + 1) = 18$.

    Consider the graph of the function f:



29. Based on the graph, $\lim_{x \to 1} f(x) =$

    (A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** DNE

    Solution: A. Using the blotter test, the limit is -1.

30. Again referring to the graph above, what is $\lim_{x \to -1} f(x) =$

    (A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** DNE

    Solution: E. Since the left and right limits differ, the limit does not exist.

31. $\lim_{x \to 0} \dfrac{x}{x^2 + 2x} =$

    (A) 0 **(B)** 1 **(C)** 1/2 **(D)** 1/3 **(E)** DNE

    Solution: C. Factor the x from the denominator and cancel it with the one in the numerator. Then the zero over zero problem disappears, and we get a limit of 1/2.

32. Let $f(x) = \dfrac{x}{2x+1}$. What is the slope of the tangent line to the graph of f at $x = 2$?

    (A) $-1/2$ **(B)** $-1/5$ **(C)** 0 **(D)** $1/25$ **(E)** $1/5$

    Solution: D. Use the quotient rule to find that $f'(2) = 1/25$.

33. Let $f(x) = x^3 - 12x + 1$. Which of the following is correct?

    (A) f is increasing on $(-\infty, \infty)$.

    (B) f is decreasing on $(-\infty, \infty)$.

    (C) f is increasing on $(-2, 2)$.

    (D) f is decreasing on $(-2, 2)$.

    (E) f is increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$.

    Solution: D. Build the sign chart for $f'(x)$ to see that $f'(x)$ is negative over the interval $(-2, 2)$, so f is decreasing over that interval.

34. Let $f(x) = x^3 - 3x^2 + 2x + 50$. Then f has a point of inflection at $x =$

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

    Solution: B. Since $f'(x) = 3x^2 - 6x + 2$, it follows that $f''(x) = 6x - 6$, so there is a change in sign at $x = 1$.

35. Let $f(x) = \ln(x) + x$. Which of the following is the equation of the tangent line to the graph of f at $x = 1$?

    (**A**) $y - 1 = (\frac{1}{x} + 1)(x - 1)$

    (**B**) $y - 2 = x - 1$

    (**C**) $y - 1 = 2(x - 2)$

    (**D**) $y - 2 = 2(x - 1)$

    (**E**) $y - 1 = 2(x - 1)$

Solution: E. Since $f'(x) = 1 + 1/x$, $f'(1) = 2$ and the line is given by $y - 1 = 2(x - 1)$.

36. Wacky Widgets, Inc. earns a daily profit of $P(x) = -10x^2 + 1760x - 50,000$ dollars when it produces x tons of widgets. Which of the following gives the marginal profit at a production level of 50 tons.

(A) $-50,000$ **(B)** 0 **(C)** 760 **(D)** 1000 **(E)** $13,000$

Solution: C. The marginal profit is $P'(x) = -20x + 1760$, so $P'(50) = 760$.

37. For a certain function g, it is known that $g'(x) = e^x + 2x$ and that $g(0) = 5$. Which of the following is closest to $g(2)$?

(A) 7.39 **(B)** 9.39 **(C)** 11.39 **(D)** 13.39 **(E)** 15.39

Solution: E. The function g must have the form $g(x) = e^x + x^2 + C$ and $g(0) = 1 + C = 5$ requires that $C = 4$. Thus, $g(2) = e^2 + 4 + 4 \approx 15.39$.

38. What is $\lim_{x\to\infty} \dfrac{1 + 2e^x}{e^x}$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** ∞

Solution: C. For large values of x, the 1 in the numerator is negligible. This the limit is 2. Alternatively, divide both numerator and denominator by e^x, and note that $1/e^x = e^{-x}$ has limit 0 as $x \to \infty$.

39. How many asymptotes, both horizontal and vertical, does $r(x) = \frac{(x-2)(x-1)(x^2)}{x(x^2-1)}$ have?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Solution: A. After cancelling common factors, we are left with $r(x) = \frac{x(x-2)}{x+1}$, which has one asymptotes, $x = -1$.

40. The derivative $f'(x) = 3x - 2$, and $f(2) = 5$. What is $f(1)$?

(A) 1/2 **(B)** 3/2 **(C)** 5/2 **(D)** 7/2 **(E)** 9/2

Solution: C. $f(x) = 3x^2/2 - 2x + C$ and $f(2) = 6 - 4 + C = 5$ implies $C = 3$, whence $f(1) = 3/2 - 2 + 3 = 5/2$.

41. Let $f(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ 2 & \text{if } x \geq 1. \end{cases}$

    What is $\lim_{x \to 1} f(x)$?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** DNE

    Solution: E. The left limit is 4 and the right limit is 2, so there is no limit.

42. Let $f(x) = 2x^2 - x + 3$. The minimum value of f on $[0, 1]$ is

    (A) 1.275 **(B)** 2.350 **(C)** 2.875 **(D)** 3.125 **(E)** 4.075

    Solution: C. The parabola opens upward and its vertex satisfies $x = 0.25$, so the minimum value of $[0, 1]$ is $f(0.25) = 2/16 - 1/4 + 3 = 23/8$.

43. Joe (who did not do well in his calculus course) now works long hours at Wacky Widgets. His supervisor has timed his work and has determined that, on a good day, Joe will have assembled a total of $N(t) = -t^3 + 6t^2 + 15t$ widgets t hours after starting work. At what rate is Joe assembling widgets 3 hours after starting work (on a good day)?

    (A) 0 **(B)** 24 **(C)** 27 **(D)** 66 **(E)** 72

    Solution: B. Since $N'(t) = -3t^2 + 12t + 15$, it follows that $N'(3) = -27 + 36 + 15 = 24$ Widgets.

44. You just ordered a new seedling from a seed catalog. If the seedling is 2 inches tall when you receive it and it will be growing at a rate of $2t + 1$ inches per month t months after you receive it, how tall will it be in 5 months?

    (A) 2 **(B)** 11 **(C)** 25 **(D)** 30 **(E)** 32

    Solution: E. The seedling will be $2 + \int_0^5 2t + 1\, dt = 32$.

Consider the function $f(x) = \ln[(x^2 - 9)(x^2 - 16)]$. The next three problems all refer to f.

45. Recall that $\ln(x)$ is defined precisely when $x > 0$. At which of the following points is f undefined?

    (A) 0.5 **(B)** 1.5 **(C)** 2.5 **(D)** 3.5 **(E)** 4.5

    Solution: D. The sign chart for $g(x) = (x-3)(x+3)(x-4)(x+4)$ shows $g(3.5) < 0$, so $f(3.5)$ is undefined there.

46. Which of the following is a critical point of f?

    (A) -9 **(B)** -5 **(C)** 1 **(D)** 2 **(E)** 7

    Solution: B. The derivative of f is

    $$f(x) = \frac{2x(x^2-16)+2x(x^2-9)}{(x^2-9)(x^2-16)} = \frac{2x(x-5)(x+5)}{(x^2-9)(x^2-16)},$$

    which has zeros $x = \pm 5$ and 0.

47. Which of the following is a critical point of f?

    (A) -6 **(B)** 0 **(C)** 6 **(D)** 8 **(E)** 9

    Solution: B. See the solution above.

48. What is the slope of the line tangent to $f(x) = xe^{2x}$ at the point $(1, e^2)$?

    (A) e^2 **(B)** $2e^2$ **(C)** $3e^2$ **(D)** $4e^2$ **(E)** $5e^2$

    Solution: C. By the product rule, $f'(x) = e^{2x} + 2xe^{2x}$, so $f'(1) = 3e^2$.

49. Find the growth of $g(x) = \ln(e^2 + x)$ over the interval $[2e^2, 5e^2]$.

    (A) $\ln 2$ **(B)** $\ln 3$ **(C)** $\ln 6$ **(D)** 2 **(E)** 3

    Solution: A. The growth of g is defined by $g(5e^2) - g(2e^2) = \ln(6e^2) - \ln(3e^2) = \ln 6 - \ln 3 = \ln 2$.

50. What is the minimum value that $f(x) = x^3 - 6x^2$ attains over the interval $[-1, 5]$?

    (A) 0 **(B)** 4 **(C)** -25 **(D)** -32 **(E)** -64

    Solution: D. Since f is cubic, we must examine its value at the left endpoint and the larger critical point. Since $f(-1) = -7$ and $f(4) = 64 - 96 = -32$, it follows that the minimum value of f over $[-1, 5]$ is -32.

51. What is the slope of the line tangent to $y = \sqrt{e^x + 3}$ at the point $(0, 2)$?

    (A) $1/8$ **(B)** $1/4$ **(C)** $1/2$ **(D)** 1 **(E)** -1

    Solution: B. By the chain rule, $y' = \frac{1}{2}(e^x + 3)^{-\frac{1}{2}} \cdot e^x$. Therefore, the slope we're looking for is $\frac{1}{2}(e^0 + 3)^{-\frac{1}{2}} \cdot e^0 = \frac{1}{2}4^{-1/2} \cdot 1 = 1/4$.

52. For which values of x is the line tangent to $g(x) = \sqrt{x^2+1}$ horizontal?

(A) 0 **(B)** 1 **(C)** -1 **(D)** 1/2 **(E)** There is no such x.

Solution: A. By the chain rule, $g'(x) = 2x \cdot \frac{1}{2}(x^2+1)^{-1/2}$, which has the value zero when $x = 0$.