December 14, 2010 **Name** ______________________________

The total number of points available is 232. Throughout this test, **show your work. Throughout this test, you are expected to use calculus to solve problems. Graphing calculator solutions will generally be worth substantially less credit. Circle your answer in each of the three multiple choice problems.** This exam is 7 pages long.

1. The function f is defined throughout $[-1, 1]$. Suppose $lim_{x\to 0}f(x) = 3$, what is $f(0)$?

   (A) 0 **(B)** 3 **(C)** It must be close to 3

   (D) $f(0)$ is not defined **(E)** Not enough information is given

2. If f is a differentiable increasing function and f is (choose one), then the relationship $f'(5) \leq \frac{f(5)-f(2)}{5-2} \leq f'(2)$ is certain to hold.

   (A) Positive **(B)** Negative **(C)** Concave up

   (D) Concave down **(E)** There is no way to guarantee the relationship.

3. If a function is always positive, then what must be true about its derivative?

   (A) The derivative is always positive.

   (B) The derivative is never negative.

   (C) The derivative is increasing.

   (D) The derivative is decreasing.

   (E) You can't conclude anything about the derivative.

4. (12 points) Find an equation for the line tangent to the graph of $f(x) = (1 + e^{-2x+4})^2$ at the point $(2, f(2))$.

5. (12 points) Find an equation for the line tangent to the graph of $f(x) = (x^2 + \ln(x))^2$ at the point $(1, f(1))$.

6. (12 points) A radioactive substance has a half-life of 29 years. Find an expression for the amount of the substance at time t if 30 grams were present initially.

7. (12 points) Build the sign chart for the rational function

$$r(x) = \frac{(x-2)^2 \cdot x(x+1)}{(x^2-4)(2x-7)^2}.$$

Clearly label the branch points.

8. (12 points) For what value(s) of x is the line tangent to $y = 4 - x^2$ parallel to the line $y = x$.

9. (12 points) Let $H(x) = \frac{1}{x-3} + \frac{1}{x^2-4}$. Is this a rational function? Find all the asymptotes.

10. (16 points) Let $A = (2, 0)$, $B = (10, 0)$, $C = (10, 12)$ and $D = (2, 6)$. The area of the quadrilateral $ABCD$ is 72.

    (a) Find an equation for the linear function (the line) that goes through the points C and D. Give this function the name f.

    (b) Use calculus to find the area of the region R defined as follows:

$$R = \{(x, y) : 2 \leq x \leq 10,\ 0 \leq y \leq f(x)\}$$

11. (12 points) Find the derivatives of each function. Then find the slope of the line tangent to the graph of f at the point $(1, f(1))$.

    (a) $f(x) = (x-1)^2 \cdot \ln(2x+1)$.

    (b) $f(x) = \frac{\ln(2x+1))}{x}$.

12. (16 points) Suppose $f(x)$ is a function whose second derivative is constant. In fact, $f''(x) = 2$. Also suppose $f'(1) = 5$ and $f(1) = 4$.

    (a) Find $f'(0)$.

    (b) Find $f(0)$.

    (c) Find the critical point(s) of f.

13. (36 points) Evaluate each of the following integrals using the Fundamental Theorem of Calculus (ie, antidifferentiate, then measure the growth of an antiderivative over the interval).

(a) Evaluate $\displaystyle\int_1^3 \frac{d}{dx}(x^2 + 8x - 2)\, dx$

(b) Evaluate $\displaystyle\int_0^{\sqrt{\ln(2)}} \frac{d}{dx}e^{x^2}\, dx$

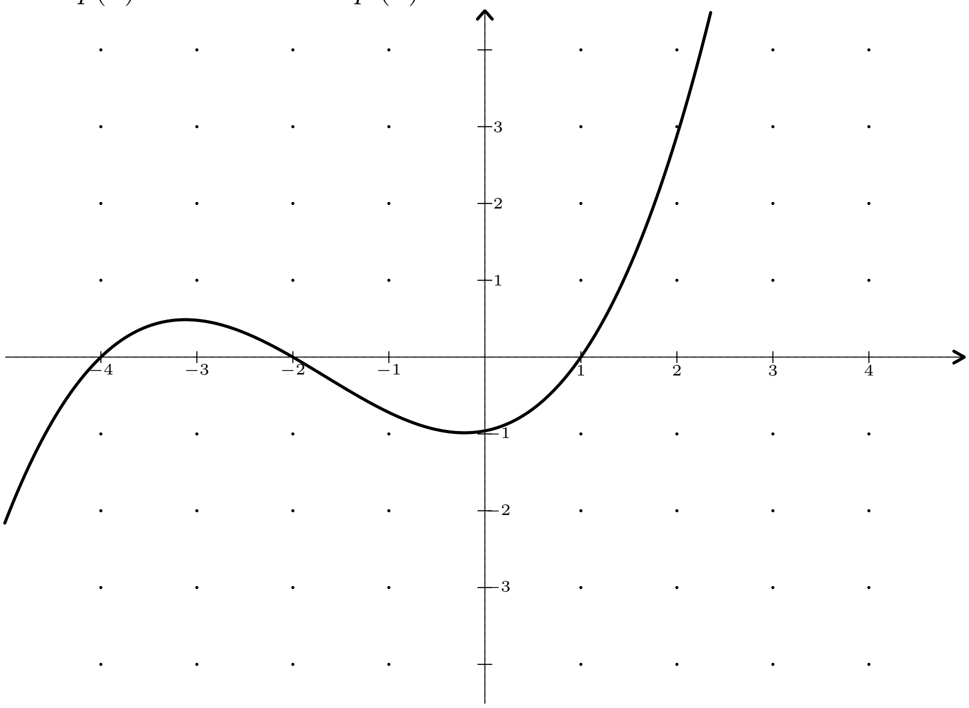
(c) Evaluate $\displaystyle\int_1^3 x^3 \cdot (x^4 - 2)^2\, dx$

(d) Evaluate $\displaystyle\int_0^4 3x^2 e^{x^3}\, dx$

14. (10 points) Evaluate $\displaystyle\int x^2 - \sqrt{x} - \frac{2}{x}\, dx$

15. (10 points) Evaluate $\displaystyle\int 3x^2\sqrt{x^3+4}\, dx$

16. (20 points) The graph of a cubic polynomial $p(x)$ is given below. On the same set of axes, sketch the graph of $p'(x)$, being especially careful about where $p(x) = 0$ and where $p'(x) = 0$.



17. (25 points) Consider the function $f(x) = \ln(3x^2 + 1)$.

    (a) Find $f'(x)$.

    (b) Find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

    (c) Find $f''(x)$.

    (d) Find the sign chart for $f''(x)$.

    (e) Find the intervals over which f is concave upwards.