December 11, 2012 **Name** ______________________________

The total number of points available is 293. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (20 points) Derivative Problem.
   Let $f(x) = 2x^2 - x$. Then $f'(x) = 4x - 1$.

   (a) Use the limit definition of derivative to verify that $f'(x) = 4x - 1$.

   (b) Use the information above to find an interval over which $f(x)$ is increasing.

2. (30 points) Limit Problem

   (a) Find $\lim_{x\to 1} \frac{x^3+x^2+3x-5}{x^2-1}$.

   (b) Suppose $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$.

      i. Is it possible that $\lim_{x\to a} f(x) \cdot g(x) = 3$?

      ii. Is it possible that $\lim_{x\to a} f(x)/g(x) = 3$?

      iii. What are the possible outcomes of $\lim_{x\to a} \frac{f(x)}{g(x)}$? Can this limit fail to exist? Must the limit fail to exist? Write a sentence or two to show that you understand this question.

3. (15 points) Consider the function $f(x) = e^{x^3-3x^2-9x}$.

   (a) Find a value of x at which the line tangent to the graph of f is horizontal.

   (b) Find an interval over which $f(x)$ is increasing.

   (c) Find an equation of the tangent line referred to in part a.

4. (30 points) There is a cubic polynomial $p(x)$ with zeros at $x = -2, x = 1$, and $x = 2$.

   (a) Build one such function.

   (b) Build the sign chart for your function.

   (c) Find an interval over which your function is increasing?

   (d) Find the area of the region bounded by your function over the interval from $x = -2$ to $x = 1$.

5. (12 points) Given $f''(x) = 2x - 6$ and $f'(-2) = 6$ and $f(-2) = 1$. Find $f'(x)$ and $f(x)$.

6. (12 points) Let $f(x) = \dfrac{3}{x} - 2e^x$.

   (a) Find an antiderivative of $f(x)$.

   (b) Compute $\displaystyle\int_1^e f(x)dx$.

7. (42 points) Find the following antiderivatives.

   (a) $\displaystyle\int 2x - 5\ dx$

   (b) $\displaystyle\int 9x^2 - 4x - 2/x\ dx$

   (c) $\displaystyle\int \frac{3x^3 + 2x^2 - x}{x}\ dx$

   (d) $\displaystyle\int \frac{2x + 3}{x^2 + 3x - 3}\ dx$

   (e) $\displaystyle\int 6x^5(x^6 + 3)^7\ dx$

   (f) $\displaystyle\int x^2 e^{x^3}\ dx$

8. (15 points) Find the intervals over which $f(x) = x^2 e^{2x}$ is increasing.

9. (12 points) Is there a value of b for which $\int_b^{2b} x^4 + x^2\,dx = 128/15$? If so, find it.

10. (30 points) Recall the $\ln(x)$ is defined for positive numbers only.

    (a) For which values of x is $g(x) = \ln\left(\frac{(x^2+1)(2x-1)}{(x^2-4)(3x+7)}\right)$ defined. Express your answer in interval notation. Notice that $g(0) = \ln\left(\frac{1(-1)}{(-4)7}\right) = \ln(1/28) = \ln(1) - \ln(28) = -\ln(28)$, so your answer above should include the number 0.

    (b) Find $g'(0)$.

    (c) Use the information above to find an equation for the line tangent to $g(x)$ at the point $(0, -\ln(28))$.

11. (20 points) Use the substitution technique to find $\int (x-2)^4 \cdot x \, dx$. Then differentiate to check your answer.

12. (10 points) Find the derivative of the function g defined by $g(x) = \ln(e^{x^2-4x})$.

13. (10 points) Compute $\int \frac{d}{dx}\, xe^{x^2}\, dx$.

14. (15 points) Suppose x and y are positive real numbers satisfying $2xy = 9$.

    (a) Find two pairs of numbers (x_1, y_1) and (x_2, y_2) satisfying the condition $2xy = 9$. Compute the value of $2x + 3y$ for each of these pairs.

    (b) What is the smallest possible value of $2x + 3y$ such that $2xy = 9$.

    (c) What is the smallest possible value of $3x + 4y$ such that $2xy = 9$.

15. (20 points) Use calculus to find the area of the trapezoid R bounded above by the graph of $f(x) = 2x + 1$, below by the x-axis, and on the sides by $x = 1$ and $x = 5$.