May 8, 2013 **Name** ________________________________

The total number of points available is 297. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (30 points) Limit Problems. Compute each of the following limits:

   (a) $\displaystyle\lim_{x\to 1}\frac{x^2+3x-4}{x^2-4x+3}$

   (b) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x^2-3}-1}{x-2}$

   (c) $\displaystyle\lim_{x\to 3}\frac{\frac{1}{x-2}-1}{x-3}$

2. (20 points) Derivative Problem.
   Let $f(x) = \sqrt{2x+1}$. Then $f'(x) = 1/\sqrt{2x+1}$. Recall that $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
   Use this limit definition of derivative to verify that $f'(x) = 1/\sqrt{2x+1}$.

3. (15 points) Consider the function $f(x) = (x + x^2 - e^{2x})^2$.

   (a) Compute $f'(x)$

   (b) Find an equation of the line tangent to the graph of f at the point $(0, f(0))$.

4. (15 points) Find an interval over which the function

$$G(x) = \ln(x^3 + x^2 + 1), \qquad -1 \leq x,$$

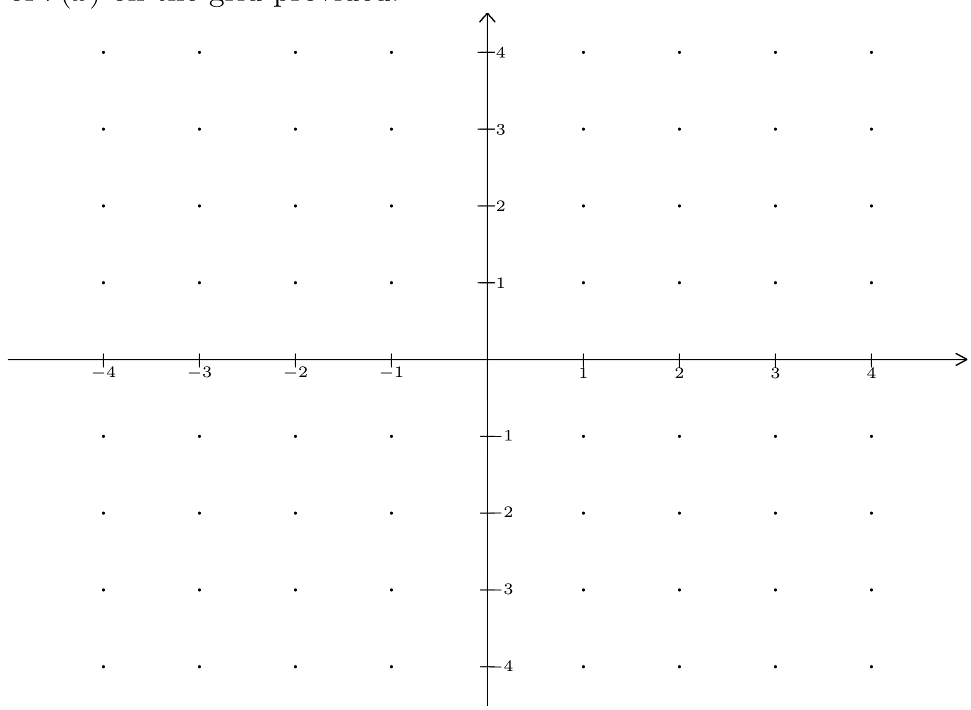
is decreasing.

5. (15 points) The function $h(x) = (3x-2)^2 \cdot x^5$ has three critical points, $x = 2/3$, $x = 0$ and a third point.

   (a) Find the third critical point.

   (b) At which of the critical points does h have a local maximum, a local minimum, or neither? In other words describe the nature of each critical point.

6. (30 points) There is a rational function $r(x)$ with exactly three zeros at $x = -2, x = 2$, and $x = 4$ and two vertical asymptotes, $x = 0$ and $x = 5$. Also, $r(x)$ has a horizontal asymptote $y = 2$. Find a symbolic representation of $r(x)$ and build the sign chart for it. The symbolic representation is not unique. Does your r have a relative max or min near 3? If so, which one. Sketch the graph of $r(x)$ on the grid provided.



7. (40 points) Let $f(x) = \dfrac{x}{2} + 1$ The region bounded by f over the interval $[0, 4]$. is a trapezoid T. Specifically, $T = \{(x, y) \mid 0 \leq x \leq 4, 0 \leq y \leq f(x)$.

   (a) Use geometry to find the area of T.

   (b) Build the Riemann sum for f over $[0, 4]$ using $n = 4$ subintervals of equal length and using the right endpoints as the sample points to determine the height of each rectangle. Is the approximation an over-estimate or an under-estimate?

   (c) Use calculus to find the area of the region T. Your calculation must show what antiderivative you used and how you measured its growth.

8. (42 points) Find the following antiderivatives.

(a) $\displaystyle\int 3x - 5\ dx$

(b) $\displaystyle\int 9x^3 - 4x - 2x^{-1}\ dx$

(c) $\displaystyle\int \frac{3x^3 + 2x^2 - 1}{x^2}\ dx$

(d) $\displaystyle\int \frac{2x}{x^2 + 3}\ dx$

(e) $\displaystyle\int 4x^3\sqrt{x^4 + 3}\ dx$

(f) $\displaystyle\int 3x^2 e^{x^3}\ dx$

9. (15 points) The percentage of alcohol in a person's bloodstream t hours after drinking 4 fluid ounces of whiskey is given by

$$A(t) = 0.24te^{-0.3t}, \qquad 0 \leq t \leq 6.$$

   (a) How fast is the percentage of alcohol in the person's bloodstream changing after 1 hour?

   (b) At what time is the percentage maximized?

   (c) What is that maximum percentage?

10. (15 points) Let $G(x) = \sqrt{x^2(2x-5)(3x+7)}$. Note that $G(3) = \sqrt{9 \cdot 1 \cdot 16} = 12$, so G is defined at 3. Find the domain of $G(x)$. Express your answer in interval notation.

11. (15 points) There is one point on the graph of the function $f(x) = \ln(x^2 + x)$ where the line tangent to the graph has a slope of 3. Find the x-coordinate of that point.

12. (15 points) Compute the number $\int_0^4 (x-2)^4 \cdot (x+2)\, dx$.

13. (30 points) A function f satisfies $f(3) = 2$. The line tangent to the graph of f at $(3, 2)$ is given by $y = 2x - 4$.

    (a) What is $f'(3)$?

    (b) Suppose that $f''(x) = x - 4$. What is $f'(2)$?

    (c) Find a representation for f.