February 6, 1998   **Name** _____

The first 10 problems count 7 points each and the final 2 count 20 points each.

1. Fill in your three character code you received via email in the box [   ]

   Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

2. What is the exact value of $|\sqrt{2} - 2| - |3 - 2\sqrt{2}|$?

   (A) 0.41   **(B)** $\boxed{\sqrt{2} - 1}$ **(C)** $1 - \sqrt{2}$ **(D)** $5 - 3\sqrt{2}$ **(E)** $3\sqrt{2} - 5$

3. Consider the function f defined by:

$$f(x) = \begin{cases} 2x^2 - 3 & \text{if } x < 0 \\ 5x - 3 & \text{if } x \geq 0 \end{cases}$$

   Find the slope of the line which goes through the points $(-2, f(-2))$ and $(3, f(3))$.

   (A) $\boxed{7/5}$ **(B)** 2   **(C)** 17/5   **(D)** 5   **(E)** 7

4. Let $f(x) = x^2 + 1$. Evaluate and simplify $\frac{f(x+h) - f(x)}{h}$.

   (A) $h - 2$ **(B)** $2x - 2h + h^2$ **(C)** $\boxed{2x + h}$

   (D) $2x + h + 2$ **(E)** $x^2 + 2h + 2$

5. Consider the function f defined by:

$$f(x) = \begin{cases} -2x + 4 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2 + 1 & \text{if } x > 1 \end{cases}$$

   The limit as x approaches 1 is

   (A) 1   **(B)** $\boxed{2}$ **(C)** 3   **(D)** 4   **(E)** The limit does not exist.

6. The expression $\dfrac{1}{1 + \sqrt{x}}$ is equivalent to

    (A) $\dfrac{1 + \sqrt{x}}{1 - x}$ **(B)** $\dfrac{1 + \sqrt{x}}{1 + x}$ **(C)** $\boxed{\frac{1 - \sqrt{x}}{1 - x}}$ **(D)** $\dfrac{1 - \sqrt{x}}{1 + x}$ **(E)** $1 + x$

7. The distance between the point $(6.5, 8.5)$ and the midpoint of the segment joining the points $(2, 3)$ and $(5, 6)$ is

    (A) $\sqrt{22}$ **(B)** $\sqrt{23}$ **(C)** $\boxed{5}$ **(D)** $\sqrt{26}$ **(E)** 6

8. The following points lie on the same line: $(2b, 3), (b + 3, -2), (b, 7)$. What is the value of b?

    (A) $-2/3$ **(B)** $2/3$ **(C)** 1 **(D)** $\boxed{4/3}$ **(E)** 2

9. Suppose the functions f and g are given completely by the table of values shown.

x	$f(x)$	x	$g(x)$
0	2	0	5
1	7	1	7
2	5	2	4
3	1	3	2
4	3	4	6
5	6	5	3
6	0	6	1
7	4	7	0

    What is $f(g(5 - 1) - 3)$?

    (A) $\boxed{1}$ **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

10. Find the **product** of the two roots of $10x^2 + 31x - 14 = 0$.

    (A) -3 **(B)** $\boxed{-7/5}$ **(C)** $5/7$ **(D)** 2 **(E)** 6

On all the following questions, **show your work.**

11. Let f and g be functions defined by $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ 4 - x & \text{if } x \geq 0 \end{cases}$
    and $g(x) = 2x + 3$.

    (a) Compute $f \circ g(-2), f \circ g(-1)$, and $f \circ g(0)$

    > $f \circ g(-2) = f(g(-2)) = f(-1) = 0$,
    > $f \circ g(-1) = f(g(-1)) = f(1) = 3$, and
    > $f \circ g(0) = f(3) = 1$.

    (b) Find a symbolic representation of $f \circ g(x)$

    > $f \circ g(x) = \begin{cases} (2x+3)^2 - 1 & \text{if } 2x + 3 < 0 \\ 4 - (2x+3) & \text{if } 2x + 3 \geq 0 \end{cases}$
    >
    > Next, simplify to get
    >
    > $$f \circ g(x) = \begin{cases} 4x^2 + 12x + 8 & \text{if } x < -3/2 \\ 1 - 2x & \text{if } x \geq -3/2 \end{cases}$$

12. Describe in English what it means to say that the limit of a function f is 2 as x approaches 1. Sketch a graph of a function which has this property but also satisfies $f(1) = 3$.

    > It means that when x is close to (but not equal to) 1, $f(x)$ is close to 2.
    >
    >