Math 1120 **Calculus** **Test 1**

July 12, 1999 Your name ____________________

On all the following questions, **show your work.**

1. (10 points) Find the <u>exact</u> value of $|\sqrt{2}-2|-|2-3\sqrt{2}|$. Leave your answer in radical form. No credit for a decimal answer.

   Solution: $|\sqrt{2}-2|-|2-3\sqrt{2}| = 2-\sqrt{2}-(3\sqrt{2}-2) = 4-4\sqrt{2}$.

2. (10 points) Find all values of x such that $-3 \le 2x-3 \le 6$.

   Solution: Add 3 to all three parts to get $-3+3 \le 2x-3+3 \le 6+3$ which is equivalent to $0 \le 2x \le 9$ which is equivalent to $0 \le x \le 9/2$.

3. (10 points) Find all roots of the equation

$$(x-1)(x+1)+(x-2)(x+1)=0.$$

   Solution: Factor $(x-1)(x+1)+(x-2)(x+1)$ to get $(x+1)((x-1)+(x-2)) = (x+1)(2x-3)=0$, which has two roots, $x=-1$ and $x=3/2$.

4. (10 points) Rationalize the numerator of the expression $\frac{\sqrt{4+h}-2}{h}$, and express your answer in simplified form.

   Solution: $\frac{\sqrt{4+h}-2}{h} = \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \frac{4+h-4}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2}$.

5. (15 points) A. What is the distance between $(-3,5)$ and $(6,8)$?

   Solution: $D = \sqrt{(-3-6)^2+(8-5)^2} = \sqrt{81+9} = 3\sqrt{10}$

   B. The points $A=(0,0), B=(8,0)$, and $C=(x,y)$ are the vertices of an equilateral triangle (i.e., all the sides have the same length). Find x and y. Write your answers in decimal form.

   Solution: Because of the symmetry, the x coordinate must be 4. The y coordinate satisfies $\sqrt{4-0)^2+(y-0)^2}=8$, which yields $y=\sqrt{48}=4\sqrt{3}$.

6. (10 points) What is the slope of the line joining the points $(-2, f(-2))$ and $(4, f(4))$, where f is the function defined by

$$f(x) = \begin{cases} x^2 - |x| & \text{if } x \le 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$$

Solution: The slope is $\frac{f(4)-f(-2)}{4-(-2)} = \frac{10-2}{6} = 4/3$.

7. (10 points) The supply function for an item is given by $p = s(x) = 0.1x^2 - 12x + 700$ and the demand function is given by $p = d(x) = 0.1x^2 + 8x - 380$, where p is measured in dollars and x is the number of items. Find the equilibrium point. That is, find the number x of items produced needed to equalize the supply and demand.

Solution: Set the two quadratics equal to one another, and notice that the second degree terms cancel to yield the linear equation $-12x + 700 = +8x - 380$ or equivalently, $20x = 1080$, so $x = 54$.

8. (40 points) Evaluate each of the limits, or state that it does not exist.

   (a) $\lim_{x\to\infty} \dfrac{x^2+9x-11}{2x^2-4x+23}$.
   Solution: The limit is just the ratio of the two coefficients of x^2, or $1/2$.

   (b) $\lim_{z\to 2} \dfrac{z^3-8}{z-2}$.
   Solution: The numerator factors into $(z-2)(z^2+2z+4)$, so the limit is just the value of (z^2+2z+4) at $z=2$, which is 12.

   (c) $\lim_{h\to 3} \dfrac{(2-h)^2+(2+h)^2}{h^2-3h+6}$.
   Solution: Just evaluate the numerator and denominator at $h=3$ to get $\frac{1^2+5^2}{9-9+6} = 26/6 = 13/3$.

   (d) $\lim_{x\to 3} \dfrac{x-3}{x^2-9}$.
   Solution: The denominator factors into $(x-3)(x+3)$, so the limit is just the value of $\frac{1}{x+3}$ at $x=3$, that is, $1/6$.

   (e) $\lim_{x\to 2} f(x)$ where

$$f(x) = \begin{cases} (x-4)^2 & \text{if } x < 2 \\ 7 & \text{if } x = 2 \\ 5x-6 & \text{if } x > 2 \end{cases}$$

   Solution: Cover the left side of the graph to find the right limit, which is the value you get from the $5x-6$ piece, namely 4. Then cover the right half to get the left limit, $\lim_{x\to 2^-}(x-4)^2$, which is also 4. Hence the limit is 4.