

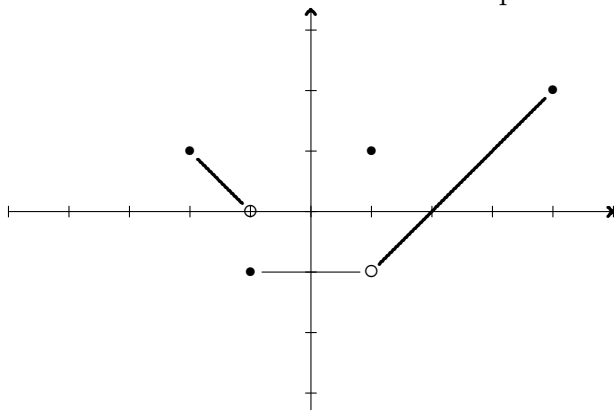
February 9, 2006

Name \_\_\_\_\_

The problems count as marked. The total number of points available is 161.

Throughout this test, **show your work.**

1. (18 points) Consider the function  $F$  whose graph is given below. Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist. The tick marks are one unit apart.



(a)  $\lim_{x \rightarrow -1^-} F(x) =$

(b)  $\lim_{x \rightarrow -1^+} F(x) =$

(c)  $\lim_{x \rightarrow -1} F(x) =$

(d)  $F(-1) =$

(e)  $\lim_{x \rightarrow 1^-} F(x) =$

(f)  $\lim_{x \rightarrow 1^+} F(x) =$

(g)  $\lim_{x \rightarrow 1} F(x) =$

(h)  $\lim_{x \rightarrow 3} F(x) =$

(i)  $F(3) =$

2. (6 points) Evaluate the limit

$$\lim_{x \rightarrow -7} \frac{x^2 + 8x + 7}{x + 7}$$

3. (6 points) Evaluate the limit

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 + 3x - 10}$$

4. (6 points) Evaluate the limit

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x^2 - 1}$$

5. (6 points) Evaluate the limit

$$\lim_{t \rightarrow 9} \frac{9 - t}{3 - \sqrt{t}}$$

6. (6 points) Evaluate the limit

$$\lim_{x \rightarrow 4} \frac{\frac{1}{x} - \frac{1}{4}}{x - 4}$$

7. (8 points) Find the midpoint of the segment joining  $(6, 3)$  and  $(-2, 7)$ . Then find the distance from that midpoint to the point  $(1, 0)$ .

8. (8 points) Let a polynomial be defined by  $p(x) = (2x - 3)^4(x - 1)(3x + 5)^3$ . What is the degree of  $p$ ? When  $p$  is written in standard form  $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$  where  $a_n \neq 0$ , what is  $a_8$ ? What is  $a_0$ ?

9. (18 points) Let

$$f(x) = \begin{cases} 9 & \text{if } x < -5 \\ -2x + 8 & \text{if } -5 \leq x < 2 \\ 0 & \text{if } x = 2 \\ 4 & \text{if } x > 2 \end{cases}$$

Sketch the graph of this function and find following limits if they exist (if not, enter DNE).

(a)  $\lim_{x \rightarrow 2^-} f(x)$

(b)  $\lim_{x \rightarrow 2^+} f(x)$

(c)  $\lim_{x \rightarrow 2} f(x)$

(d)  $\lim_{x \rightarrow -5^-} f(x)$

(e)  $\lim_{x \rightarrow -5^+} f(x)$

(f)  $\lim_{x \rightarrow -5} f(x)$

10. (12 points) Consider the function whose properties are displayed.

$a$	-1	0	1	2	3	4
$\lim_{x \rightarrow a^-} f(x)$	DNE	1	1	3	2	3
$\lim_{x \rightarrow a^+} f(x)$	1	1	1	3	2	DNE
$f(a)$	1	1	-1	3	2	3
$\lim_{x \rightarrow a^-} g(x)$	DNE	1	3	3	1	0
$\lim_{x \rightarrow a^+} g(x)$	1	2	3	3	1	DNE
$g(a)$	1	-1	3	3	1	0

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a)  $\lim_{x \rightarrow -1^-} [f(x) + g(x)]$

(b)  $\lim_{x \rightarrow 3} [f(x) + g(x)]$

(c)  $f(1)g(1)$

(d)  $f(2) + g(0)$

11. (6 points) Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{2 + 4x}{9 - 2x}$$

12. (6 points) Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{2x^3 - 10x^2 - 3x}{7 - 6x - 10x^4}$$

13. (8 points) Find the (implied) domain of

$$f(x) = \frac{\sqrt{x-7}}{(x-2)(x-9)},$$

and write your answer in interval notation.

14. (8 points) Find all the  $x$ -intercepts of the function

$$g(x) = 3(2x - 7)^3(2x + 1)^2 - 6(2x - 7)^2(2x + 1)^3.$$

15. (8 points) Compute the exact value of  $|6\pi - 10\sqrt{2}| + |6\pi - 20| - |5\sqrt{2} - 8|$ .  
No points for a decimal approximation.

16. (8 points) Find an equation for a line perpendicular to the line  $2x - 5y = 11$  and which goes through the point  $(-2, 6)$ .

17. (8 points) Suppose  $f(x) = \sqrt{3x - 1}$  and  $g(x) = x^2 + 4$ . Find the two composite functions

(a)  $f \circ g(x)$

(b)  $g \circ f(x)$

18. (15 points) Let  $f(x) = \sqrt{2x - 1}$ .

(a) Find the slope of the line joining the points  $(5, 3)$  and  $(x, f(x))$ , where  $x \neq 5$ .

(b) Then find the limit of the expression in (a) as  $x \rightarrow 5$ . Call this limit  $f'(5)$ .

(c) Use the information found in (b) to write an equation for the line tangent to the graph of  $f$  at the point  $(5, 3)$ .