February 12, 2014 **Name** _____

The problems count as marked. The total number of points available is 174. Throughout this test, **show your work.**

1. (10 points) A line L is given by the equation $2x + 3y = 6$. Another line L' perpendicular to L passes through the point $(2, 5)$. Find the y-intercept of L'. Then find the x-intercept of L'.

2. (10 points) Find all solutions to $||3x - 5| - 3| = 4$.

3. (10 points) Find the exact value of the expression

$$|3\pi - 8| + |2\pi - 4| + |5\pi - 17|.$$

   Use the symbol π in your answer if you need to.

4. (10 points) What is the distance from the center of the circle $x^2 + y^2 + 4y = 21$ to the point $(3, 2)$? Is the point $(3, 2)$ **inside**, **outside**, or **on** the the circle?

5. (30 points) Evaluate each of the limits indicated below.

(a) $\lim_{x\to-\infty} \dfrac{3x^4-6}{(11-3x^2)^3}$

(b) $\lim_{x\to 1} \dfrac{(x+1)^2-4}{(x+2)^2-9}$

(c) $\lim_{h\to 0} \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$.

(d) $\lim_{x\to\infty} \dfrac{(2x^2+3)^3}{(3x^3+x-2)^2}$

(e) $\lim_{h\to 0} \dfrac{\sqrt{25+2h}-5}{h}$

(f) $\lim_{x\to 3} \dfrac{x^3-27}{x-3}$

6. (12 points) The points $(1, 0), (5, 1), (u, v)$, and $(0, 4)$ are the vertices of a square. Find u and v.

7. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{x^2 - 2x - 3}}{x - 9}.$$

Express your answer as a union of intervals. That is, use interval notation.

8. (12 points) Let $H(x) = (x^2-4)^2(x-3)^3$. Using the chain rule and the product rule,

$$H'(x) = 2(x^2 - 4) \cdot 2x(x - 3)^3 + (x^2 - 4)^2 \cdot 3(x - 3)^2.$$

Find all five zeros of $H'(x)$.

9. (21 points) Let

$$f(x) = \begin{cases} 2x+3 & \text{if } -1 < x \leq 0 \\ |x-3| & \text{if } 0 < x < 4 \\ 2 & \text{if } x = 4 \\ 5-x & \text{if } 4 < x \leq 6 \end{cases},$$

(a) What is the domain of f? Express your answer in interval notation.

(b) What is $\lim_{x \to 0^-} f(x)$?

(c) What is $\lim_{x \to 0^+} f(x)$?

(d) Is f continuous at $x = 0$? Discuss why or why not.

(e) What is $\lim_{x \to 4^-} f(x)$?

(f) What is $\lim_{x \to 4^+} f(x)$?

(g) Is f continuous at $x = 4$? Discuss why or why not.

10. (20 points) Let $f(x) = \sqrt{3x-2}$.

    (a) Let h be a positive number. What is the slope of the line passing through the points $(6, f(6))$ and $(6+h, f(6+h))$. Your answer depends on h, of course. Suppose your answer is called $G(h)$.

    (b) Compute $\lim_{h \to 0} G(h)$.

    (c) Your answer to (2) is the slope of the line tangent to the graph of f at the point $(6, f(6))$. In other words, your answer is $f'(6)$. Write and equation for the tangent line.

11. (12 points) Let $f(x) = (2x-3)^5(5x^2-1)+17x^5$, let $g(x) = (x-4)^4(8x^3)-2x^4$.

    (a) What is the degree of the polynomial $f - g$?

    (b) What is the degree of the polynomial $f \cdot g$?

    (c) Estimate within one tenth of a unit the value of $f(10000)/g(10000)$.

    (d) Compute $\lim_{x\to\infty} \frac{f(x)}{g(x)}$.

12. (15 points) Recall that the Intermediate Value Theorem guarantees that for any function f continuous over the interval $[a, b]$ and for any number M between $f(a)$ and $f(b)$, there exists a number c such that $f(c) = M$. The function $f(x) = \frac{1}{1+\frac{1}{x}}$ is continuous for all $x > 0$. Let $a = 1$.

    (a) Pick a number $b > 1$ (any choice is right), and then find a number M between $f(a)$ and $f(b)$.

    (b) Show that the conclusion to the Intermediate Value Theorem is satisfied by finding a number c in (a, b) such that $f(c) = M$.