Math 1120 **Calculus** **Test 1.**

January 31, 2001 **Name** ______________________

The first 8 problems count 7 points each and the final 4 count as marked.

1. Fill in the three character code you received via email in the box $\boxed{}$

   Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

2. Which of the following numbers belong to the (implied) domain of

$$f(x) = \frac{\sqrt{x-2}}{x-4}?$$

   Circle all those that apply.

   (A) -2 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

   Solution: The domain includes all real numbers greater than or equal to 2 except 4, which makes the denominator zero. Thus, 2, 3, and 5 all belong to the domain.

3. What is the y-intercept of the line defined by $\frac{x}{3} + \frac{y}{6} = 2$?

   (A) -2 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 16

   Solution: The y-intercept is the point on the line for which $x = 0$. Solving for y gives $y = 12$.

4. Let $f(x) = 2x + 3$ and $g(x) = 3x - 3$. Which of the following does not belong to the domain of f/g?

   (A) 1 **(B)** 3 **(C)** 6 **(D)** 9 **(E)** The domain of f/g is the set of all real numbers.

   Solution: Only a number for which g is zero fails to be in the domain. Solving $3x - 3 = 0$ yields $x = 1$.

5. Referring to the f and g of the previous problem, what is the value of $g(f(g(3)))$?

   (A) -3 **(B)** 15 **(C)** 42 **(D)** 45 **(E)** 54

   Solution: $g(f(g(3))) = g(f(9-3)) = g(2 \cdot 6 + 3) = 3 \cdot 15 - 3 = 42.$

6. Let $f(x) = x^2 + 1$. Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$.

   (A) $h - 2$ **(B)** $2x - 2h + h^2$ **(C)** $2x + h$

   (D) $2x + h + 2$ **(E)** $x^2 + 2h + 2$

   Solution: Simplify $\frac{(x+h)^2+1-(x^2+1)}{h}$ to get $\frac{x^2+2xh+h^2+1-x^2-1)}{h} = \frac{2xh+h^2}{h}$, whereupon, the h can be factored from the numerator and cancelled with the h in the denominator to yield $2x + h$.

   Suppose the functions f and g are given completely by the table of values shown.

x	$f(x)$	x	$g(x)$
0	2	0	5
1	7	1	7
2	5	2	4
3	1	3	2
4	3	4	6
5	6	5	3
6	0	6	1
7	4	7	0

7. Solve the equation $f \circ g(x) = 7$?

   (A) 1 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

   Solution: Since $f(1) = 7$, it must be the case that $g(x) = 1$. This is true only when $x = 6$.

8. Compute $(f \cdot g)(g(3))$?

   (A) 18 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 30

   Solution: $(f \cdot g)(g(3)) = f(2) \cdot g(2) = 5 \cdot 4 = 20$.

   On all the following questions, **show your work.**

9. (20 points) Let f and g be functions defined by $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ 4 - x & \text{if } x \geq 0 \end{cases}$ and $g(x) = 2x + 3$.

   (a) Compute $g \circ f(-1)$, $g \circ f(0)$, and $g \circ f(1)$
   Solution: Note that $g \circ f(-2) = g(3) = 9$, $g \circ f(-1) = g(0) = 3$, $g \circ f(0) = g(4) = 11$, and $g \circ f(1) = g(3) = 9$

   (b) Find a symbolic representation of $g \circ f(x)$

Solution: $g \circ f(x) = \begin{cases} 2(x^2-1)) + 3 & \text{if } x < 0 \\ 2(4-x)+3 & \text{if } x \geq 0 \end{cases}$

Next, simplify to get

$$g \circ f(x) = \begin{cases} 2x^2+1 & \text{if } x < 0 \\ 11-2x & \text{if } x \geq 0 \end{cases}$$

10. (10 points) Cowling's Rule can be used to calculate drug doses for children. If a is the adult dosage and t is the age of the child in years, then the child's dosage is

$$D(t) = \left(\frac{t+1}{24}\right) a.$$

If the adult dosage for a certain drug is 300 mg and the child is 5 years old, how much drug should be prescribed?

Solution: $D(5) = \frac{5+1}{24} \cdot 300 = 75.$

11. (25 points) Compute each of the following limits.

(a) Let $f(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

$\lim_{x \to 1} f(x)$

Solution: Use the blotter test to see that $f(x)$ is close to 3 when x is close (but not equal) to 1.

(b) $\lim_{x \to 2} \dfrac{x^2-4}{x-2}$

Solution: Factor the numerator and cancel out the factor $x-2$ to get $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{x+2}{=} 4.$

(c) $\lim_{x \to 1} \dfrac{x-1}{x^3-1}$

Solution: Factor the denominator and cancel out the factor $x-1$ to get $\lim_{x \to 1} \frac{1}{x^2+x+1} = 1/3.$

(d) $\lim_{x \to 3} 2x^3\sqrt{x^2+7}$

Solution: Just replace all the x's with the number 3 to get $2 \cdot 3^3\sqrt{9+7} = 54 \cdot 4 = 216.$

(e) $\lim_{x \to \infty} \dfrac{2x^2}{1+x^2}$

Solution: We are looking for the horizontal asymptote, which by the asymptote theorem is just $2/1 = 2$.

12. (15 points) Describe in English what it means to say that "the limit of a function f is 2 as x approaches 1". Sketch a graph of a function which has this property but also satisfies $f(1) = 3$.

    Solution: It means that when x is close to, but not equal to, 1, $f(x)$ is close (and possibly equal) to 2. A function with the desired properties is given below.