Math 1120   **Calculus, sections 1 and 2**   **Test 1**

February 11, 2015   **Name** _____

The problems count as marked. The total number of points available is 150. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit. Note please that this test is a composite of the tests for sections 1 and 2.

1. (6 points) Use the definition of absolute value to find the exact value of $|3\pi - 10 - \sqrt{2}| + |4 - \sqrt{2}|$. You might find it necessary to use the symbols π and/or $\sqrt{2}$.

2. (10 points) Five hikers A, B, C, D and E recorded their distance hiked and time or various trails. List the hikers in order from slowest to fastest. Also, how much faster is the fastest hiker than the slowest hiker?

Hours
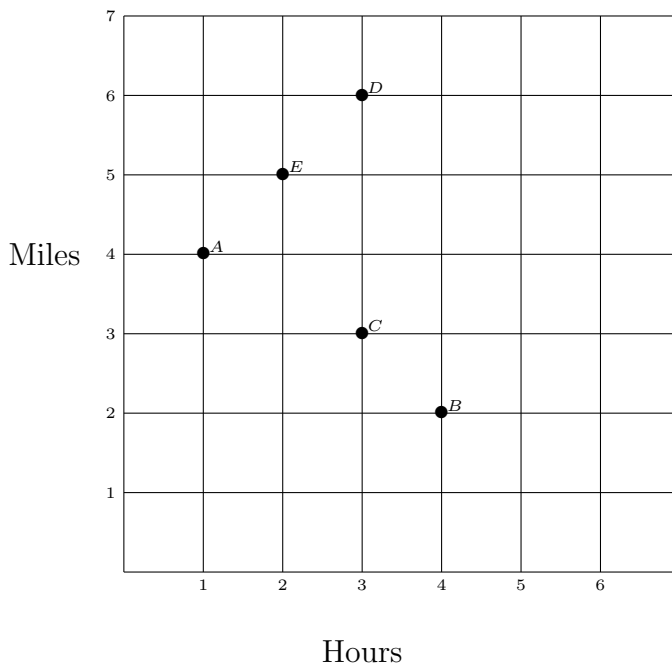
3. (12 points) Let $A = (1, 2)$ and $B = (4, 6)$ be two points in the plane.

   (a) Find an equation for the line passing through both A and B.

   (b) Find an equation for the circle centered at A and passing through B.

   (c) Find the midpoint of the line segment joining A and B.

4. (59 points) Evaluate each of the limits (and function values) indicated below. It is very important to show your work on these problems. A correct 'naked' answer is worth 1 point.

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(a) $\lim\limits_{x \to 1} \dfrac{x^3 + x^2 + x - 3}{x^3 - 3x^2 + 5x - 3}$

(b) $\lim\limits_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

(c) $\lim\limits_{x \to 7} \dfrac{\sqrt{x - 3} - 2}{x - 7}$

(d) $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{x^3 + 1}$

(e) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^3 - 1}$

(f) $\lim\limits_{h \to 0} \dfrac{(2 + h)^3 - 8}{h}$.

(g) $\lim\limits_{x \to \infty} \dfrac{\sqrt{16x^2 - 3}}{11 - 5x}$

(h) $\lim\limits_{x \to \infty} \dfrac{6x^5 - 3x^3}{11 - 12x^4}$

(i) $\lim\limits_{x \to \infty} \dfrac{6x^5 - 3x^3}{11 - 12x^5}$

The following 10 problems are worth 1 point each. For problems below, let

$$f(x) = \begin{cases} 0 & \text{if } -3 < x < 0 \\ x - 1 & \text{if } 0 \leq x < 2 \\ -1 & \text{if } x = 2 \\ 1 - x & \text{if } x > 2 \end{cases}$$

Find the value, if it exists, of each item below. Use DNE when the limit does not exist.

(j) What is the domain of the function f?

(k) $\displaystyle\lim_{x \to 0^-} f(x)$

(l) $\displaystyle\lim_{x \to 0^+} f(x)$

(m) $\displaystyle\lim_{x \to 0} f(x)$

(n) $f(0)$

(o) $\displaystyle\lim_{x \to 2^-} f(x)$

(p) $\displaystyle\lim_{x \to 2^+} f(x)$

(q) $\displaystyle\lim_{x \to 2} f(x)$

(r) $f(2)$

(s) Is f continuous at $x = 0$?

5. (10 points) Find all the x-intercepts of the function

$$g(x) = 2(x-1)(2x+1)^2 + (x-1)^2(2x+1).$$

6. (15 points)

(a) Find all solutions of the inequality $\mid 3x - 7 \mid \leq 5$ and write your solution in interval notation.

(b) Find the (implied) domain of

$$f(x) = \sqrt{\mid 3x - 7 \mid - 5},$$

and write your answer in interval notation.

7. (20 points) Let $f(x) = \sqrt{2x+1}$. Notice that $f(4) = \sqrt{2 \cdot 4 + 1} = 3$.

   (a) Find the slope of the line joining the two points $(4, f(4))$ and $(5, f(5))$.

   (b) Let h be a positive number. What is the slope of the line passing through the points $(4, f(4))$ and $(4 + h, f(4 + h))$. Your answer depends on h, of course.

   (c) Compute $\lim_{h \to 0} \frac{f(4+h)-f(4)}{h}$ to get $f'(4)$.

   (d) Your answer to (c) is the slope of the line tangent to the graph of f at the point $(4, f(4))$. In other words, your answer is $f'(4)$. Write and equation for the tangent line.

8. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

   (a) Find the slope of the line joining the points $(0, 1)$ and $(0 + h, f(0 + h)) = (h, f(h))$, where $h \neq 0$.

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   (c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(0, 1)$.

9. (12 points) Two circles C_1 and C_2 are given, $C_1 : x^2 + 4x + y^2 - 6y = 12$ and $C_2 : x^2 + y^2 - 2y = 0$.

    (a) What is the distance between the centers of the two circles.

    (b) Find an equation for the line joining the centers of the two circles.

    (c) How many points belong to both circles?

    (d) What is the distance from the point $P = (3, 5)$ to the point on C_1 that is closest to P?

10. (12 points) The midpoints of the segments AB joining $A = (1, 3)$ and $B = (-1, 7)$ and CD joining $C = (-2, 4)$ and $D = (4, 6)$ are joined by a line L.

    (a) What is the slope of the line L.

    (b) How far apart are the two midpoints?

    (c) Find an equation for the line perpendicular to L and passing through the midpoint of the segment AB.

11. (12 points) Consider the parabola defined by $y = x^2 - 3x + 1$.

    (a) Write the equation in vertex form $y = a(x - h)^2 + k$ to find the vertex of the parabola.

    (b) Use the information in (a) to find the smallest value of y among all the points on the parabola.

12. (12 points) The vertices of a square are $(0, 1), (4, 4), (7, 0)$ and (u, v).

    (a) What is the area of the square?

    (b) What are the coordinates u and v?