**Math 1120 Calculus Test 1**

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**June 4, 2001** **Name** ______________________________

The first 9 problems count 6 points for each part and the final 4 count as marked. The total number of points possible is 127.

1. What is the $y$-intercept of the line passing through the points $(4, 7)$ and $(8, 2)$?

   **Solution:** The slope is $\frac{2-7}{8-4} = \frac{-5}{4}$ so the line in question has point-slope form $y - 7 = -\frac{5}{4}(x - 4)$ which in slope-intercept form is $y = -\frac{5}{4}x + 12$ so the $y$-intercept is 12.

2. What is the exact value of $|2\sqrt{7} - 5| - |7 - 3\sqrt{7}|$?

   **Solution:** Because $2\sqrt{7} - 5$ is positive, $|2\sqrt{7} - 5| = 2\sqrt{7} - 5$ and because $|7 - 3\sqrt{7}|$ is negative, $|7 - 3\sqrt{7}| = -(7 - 3\sqrt{7}) = 3\sqrt{7} - 7$. Therefore, $|2\sqrt{7} - 5| - |7 - 3\sqrt{7}| = 2\sqrt{7} - 5 - 3\sqrt{7} + 7 = 2 - \sqrt{7}$

3. Express the value of $6^9 \cdot 9^6 \cdot 6^6 \cdot 9^9$ in the form $a^b$.

   **Solution:** $6^9 \cdot 9^6 \cdot 6^6 \cdot 9^9 = 6^{15} \cdot 9^{15} = 54^{15}$.

4. Consider the function $f$ defined by:

$$f(x) = \begin{cases} 2x^2 - 7 & \text{if } x < 0 \\ 5x - 1 & \text{if } x \geq 0 \end{cases}$$

   Find the slope of the line which goes through the points $(-2, f(-2))$ and $(3, f(3)$.

   **Solution:** The slope is $\frac{14-1}{3-(-2)} = \frac{13}{5}$.

5. Consider the function $f$ defined by:

$$f(x) = \begin{cases} -2x + 5 & \text{if } x < 1 \\ 5 & \text{if } x = 1 \\ x^2 + 2 & \text{if } x > 1 \end{cases}$$

   Find $\lim_{x \to 1} f(x)$.

   **Solution:** By the blotter test or by algebra, the limit is 3.

6. The expression $\dfrac{1}{1+\sqrt{x}}$ is equivalent to

   **(A)** $\dfrac{1+\sqrt{x}}{1-x}$ **(B)** $\dfrac{1+\sqrt{x}}{1+x}$ **(C)** $\dfrac{1-\sqrt{x}}{1-x}$ **(D)** $\dfrac{1-\sqrt{x}}{1+x}$ **(E)** $1+x$

   **Solution:** C. Rationalize the numerator by multiplying by the fraction $1 = \frac{1-\sqrt{x}}{1-\sqrt{x}}$ to get $\frac{1-\sqrt{x}}{1-x}$.

7. What is the distance between the point $(4.5, 10.5)$ and the midpoint of the segment joining the points $(2, 4)$ and $(5, 7)$?

   **Solution:** The distance is $d = \sqrt{(3.5-4.5)^2 + (5.5-10.5)^2} = \sqrt{26}$.

8. Suppose the functions $f$ and $g$ are given completely by the table of values shown.

| $x$ | $f(x)$ | $x$ | $g(x)$ |
|---|---|---|---|
| 0 | 2 | 0 | 5 |
| 1 | 7 | 1 | 7 |
| 2 | 5 | 2 | 4 |
| 3 | 1 | 3 | 2 |
| 4 | 3 | 4 | 6 |
| 5 | 6 | 5 | 3 |
| 6 | 0 | 6 | 1 |
| 7 | 4 | 7 | 0 |

   (a) What is $(f \div g)(5-1)$?
   **Solution:** $(f \div g)(5-1) = f(4)/g(4) = 3/6 = 1/2$.

   (b) What is $f(g(5)+3)$?
   **Solution:** $f(g(5)+3) = f(6) = 0$.

   (c) Find a value of $x$ such that $g(f(x)) = 6$.
   **Solution:** Since $g(4) = 7$, we must find an $x$ for which $f(x) = 4$. $x = 7$ does the trick.

   (d) What is $(g \circ f)(g(2) - f(3))$?
   **Solution:** $(g \circ f)(g(2) - f(3)) = g \circ f(4-1) = g(f(3) = g(1) = 7$.

9. Find the **product** of the two roots of $6x^2 + 70x - 24 = 0$.

   **Solution:** Notice that $6x^2+70x-24 = 0$ can be factored into $2(3x-1)(x+12)$ so the roots are $x = 1/3$ and $x = -12$, the product of which is $-4$.

10. (10 points) Let $f(x) = x^2 - x$. Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$.

    **Solution:** Notice that $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-(x+h)-(x^2-x)}{h} = \frac{x^2+2xh+h^2-x-h-x^2+x}{h}$ $= \frac{2xh+h^2-h}{h} = \frac{h(2x+h-1)}{h} = 2x + h - 1$.

11. (15 points) Let $f$ and $g$ be functions defined by $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ 4 - x & \text{if } x \geq 0 \end{cases}$ and $g(x) = 2x + 3$.

    (a) Compute $f \circ g(-2)$,$f \circ g(-1)$, and $f \circ g(0)$
    **Solution:** $f \circ g(-2) = f(g(-2)) = f(-1) = 0$,
    $f \circ g(-1) = f(g(-1)) = f(1) = 3$, and
    $f \circ g(0) = f(3) = 1$.

    (b) Find a symbolic representation of $f \circ g(x)$
    **Solution:** $f \circ g(x) = \begin{cases} (2x+3)^2 - 1 & \text{if } 2x + 3 < 0 \\ 4 - (2x+3) & \text{if } 2x + 3 \geq 0 \end{cases}$
    Next, simplify to get

$$f \circ g(x) = \begin{cases} 4x^2 + 12x + 8 & \text{if } x < -3/2 \\ 1 - 2x & \text{if } x \geq -3/2 \end{cases}$$

12. (20 points) Compute the following limits.

    (a) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$
    **Solution:** Factor the numerator and cancel out the factor $x - 2$ to get $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2} = \lim\limits_{x \to 2} \dfrac{x + 2}{1} = 4$.

    (b) $\lim\limits_{x \to 1} \dfrac{x - 1}{x^3 - 1}$
    **Solution:** Factor the denominator and cancel out the factor $x - 1$ to get $\lim\limits_{x \to 1} \dfrac{1}{x^2 + x + 1} = 1/3$.

    (c) $\lim\limits_{x \to 1} 2x^3\sqrt{2x + 7}$
    **Solution:** Just replace all the $x$'s with the number 1 to get $2 \cdot 1^3 \sqrt{2 + 7} = 2 \cdot 3 = 6$.

(d) $\lim_{x\to\infty} \dfrac{2x^2}{1+x^2}$
**Solution:** We are looking for the horizontal asymptote, which by the asymptote theorem is just $2/1 = 2$.

13. (10 points) Describe in English what it means to say that the limit of a function $f$ is 3 as $x$ approaches 2. Sketch a graph of a function which has this property but also satisfies $f(3) = 1$.

    **Solution:** It means that when $x$ is close to (but not equal to) 2, $f(x)$ is close to 3.