October 14, 2015 **Name** _____

The problems count as marked. The total number of points available is 171. Throughout this test, **show your work.** Use of calculator to circumvent ideas discussed in class will generally result in no credit.

1. (20 points)

   (a) Find all solutions to $||x-3|-8|=5$.

   (b) Find the domain of the function $f(x)=\sqrt{||x-3|-8|-5}$ and write your answer in interval form.

2. (24 points) The set of points C_1 in the plane satisfying $x^2+y^2=4$ is a circle. The set C_2 whose points satisfy $x^2-16x+y^2-12y=-36$ is also a circle.

   (a) What is the distance between the centers of the circles?

   (b) How many points in the plane belong to both circles. That is, how many points in the plane satisfy both equations?

   (c) Find an equation for the line connecting the centers of the circles.

3. (35 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to 3} \dfrac{x^2-6x+9}{x^2+x-12}$

   (b) $\lim_{x\to 2} \dfrac{x^3-8}{x^3-4x^2+7x-6}$

   (c) $\lim_{x\to 4} \dfrac{x-4}{x^3-64}$

   (d) $\lim_{x\to -3} \dfrac{x^3+27}{x+3}$

   (e) $\lim_{x\to 2} \dfrac{\frac{1}{3x}-\frac{1}{6}}{x-2}$

   (f) $\lim_{x\to 5} \dfrac{\sqrt{4x+5}-5}{x-5}$

   (g) $\lim_{x\to \sqrt{8}} \dfrac{x^4-64}{x^2-8}$

   (h) $\lim_{x\to \infty} \dfrac{(2x-3)^2(3x+1)}{(6x-1)^3}$

4. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{(x^2-16)(2x-3)}}{x^2-9}.$$

   Express your answer as a union of intervals. That is, use interval notation.

5. (12 points) Let $H(x) = (x^2-9)^2(2x-3)^2$. Using the chain rule and the product rule,

$$H'(x) = 2(x^2-9)\cdot 2x(2x-3)^2 + 2(x^2-9)^2\cdot 2(2x-3).$$

   Three of the zeros of $H'(x)$ are $x = \pm 3$ and $x = 3/2$. Find the other two.

6. (15 points) Let $f(x) = (x^2 - 4)^4$

   (a) Find $f'(x)$

   (b) Use the information you found in (a) to find an equation for the line tangent to f at the point $(3, 625)$.

   (c) Find all the critical points of f.

7. (18 points) If a ball is shot vertically upward from the roof of 128 foot building with a velocity of 256 ft/sec, its height after t seconds is $s(t) = 128+256t-16t^2$.

   (a) What is the height the ball at time $t = 1$?

   (b) What is the velocity of the ball at the time it reaches its maximum height?

   (c) What is the maximum height the ball reaches?

   (d) After how many seconds is the ball exactly 374 feet above the ground?

   (e) How fast is the ball going the first time it reaches the height 374 feet?

   (f) How fast is the ball going the second time it reaches the height 374 feet?

8. (10 points) The demand curve for a new phone is given by $3p + 2x = 18$ where p is the price in hundreds of dollars and x is the number demanded in millions. The supply curve is given by $x - p^2 + 4p = 3$. Find the point of equilibrium.

9. (25 points) Let $f(x) = \sqrt{x^2 - 5}$.

   (a) Let h be a positive number. What is the slope of the line passing through the points $(3, f(3))$ and $(3 + h, f(3 + h))$. Your answer depends on h, of course. Suppose your answer is called $G(h)$.

   (b) Compute $\lim_{h \to 0} G(h)$.

   (c) What is $f'(3)$?

   (d) Write an equation for the tangent line at $x = 3$.