

October 3, 2016

Name _____

The problems count as marked. The total number of points available is 171. Throughout this test, **show your work**. This is an amalgamation of the tests from sections 3 and 10.

1. (10 points) Find an equation for a line parallel to the line $2y + 3x = 12$ which passes through the point $(3, 5)$.

2. (20 points) Write the set of points that satisfy $||2x - 15| - 3| \leq 2$ using interval notation.

3. (36 points) Evaluate each of the limits indicated below.

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

(b) $\lim_{x \rightarrow 1} \frac{x^3 - 5x^2 + 7x - 3}{x^3 - 3x + 2}$ Hint: think about why this is a zero over zero problem.

(c) $\lim_{x \rightarrow 2} \frac{\frac{1}{3x} - \frac{1}{6}}{x - 2}$

(d) $\lim_{x \rightarrow 6} \frac{\sqrt{6x} - 6}{x - 6}$

(e) $\lim_{x \rightarrow -\infty} \frac{(2 - x)(10 + 6x)}{(3 - 5x)(8 + 8x)}$

(f) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^6 - 6}}{x^2 + 6}$

4. (24 points) The set of points C_1 in the plane satisfying $x^2 + y^2 - 6y = 0$ is a circle. The set C_2 whose points satisfy $x^2 - 12x + y^2 - 8y = -48$ is also a circle.

(a) What is the distance between the centers of the circles?

(b) How many points in the plane belong to both circles. That is, how many points in the plane satisfy both equations?

(c) Find an equation for the line connecting the centers of the circles.

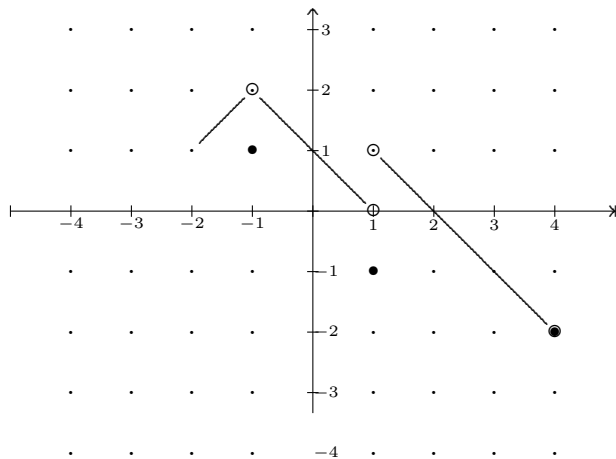
5. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{(x+4)(2x-3)(3x-17)}}{x-6}.$$

Express your answer as a union of intervals. That is, use interval notation.

6. (12 points) Let $H(x) = (x+1)(x^2-9) - (x-3)(3x+5)$. Find the zeros of the function.

7. (18 points) Consider the function F whose graph is given below. Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist. The tick marks are one unit apart.



(a) $\lim_{x \rightarrow -1^-} F(x) =$

(b) $\lim_{x \rightarrow -1^+} F(x) =$

(c) $\lim_{x \rightarrow -1} F(x) =$

(d) $F(-1) =$

(e) $\lim_{x \rightarrow 1^-} F(x) =$

(f) $\lim_{x \rightarrow 1^+} F(x) =$

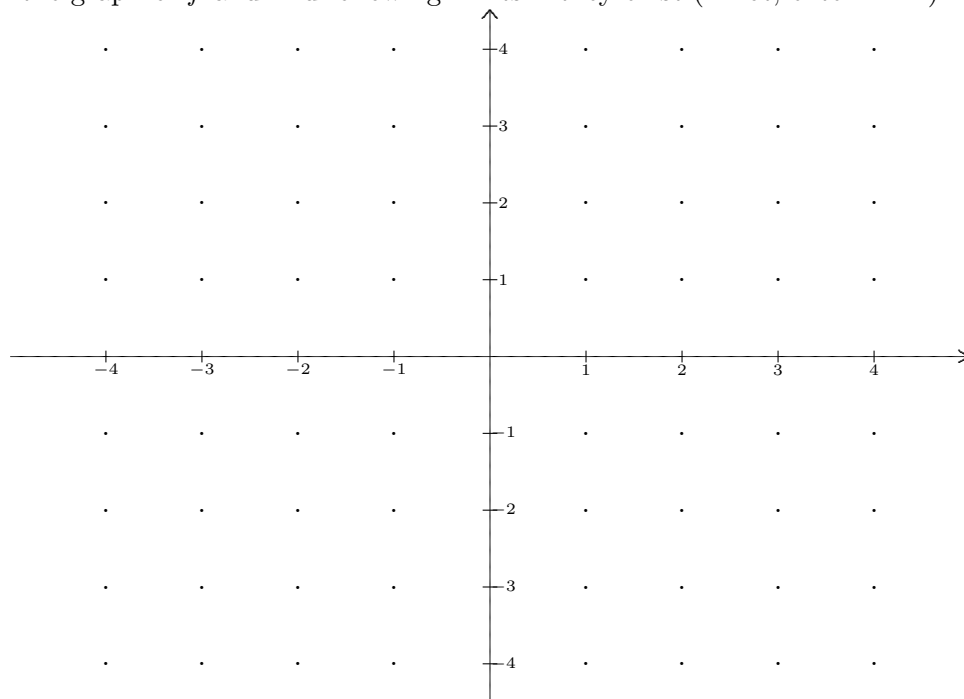
(g) $\lim_{x \rightarrow 1} F(x) =$

(h) $\lim_{x \rightarrow 3} F(x) =$

(i) $F(3) =$

8. (18 points)

$$f(x) = \begin{cases} 3 & \text{if } 2 < x \leq 4 \\ 4 & \text{if } x = 2 \\ -x + 1 & \text{if } 0 \leq x < 2 \\ x + 1 & \text{if } -4 \leq x < 0 \end{cases}$$

Sketch the graph of f and find following limits if they exist (if not, enter DNE).(a) Express the domain of f in interval notation.

(b) $\lim_{x \rightarrow 2^-} f(x)$

(c) $\lim_{x \rightarrow 2^+} f(x)$

(d) $\lim_{x \rightarrow 2} f(x)$

(e) $\lim_{x \rightarrow 0^-} f(x)$

(f) $\lim_{x \rightarrow 0} f(x)$

9. (12 points) Let $f(x) = (2x^2 - 3)^3(5x - 1) + 17x^5$, let $g(x) = (3x - 4)(2x^3)^2 - 2x^4$.

(a) What is the degree of the polynomial $f + g$?

(b) What is the degree of the polynomial $f \cdot g$?

(c) Estimate within one tenth of a unit the value of $f(10000)/g(10000)$.

(d) Compute $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$.

10. (15 points) Recall that the Intermediate Value Theorem guarantees that for any function f continuous over the interval $[a, b]$ and for any number M between $f(a)$ and $f(b)$, there exists a number c such that $f(c) = M$. The function $f(x) = \frac{1}{1 + \frac{1}{x}}$ is continuous for all $x > 0$. Let $a = 1$.

(a) Pick a number $b > 1$ (any choice is right), and then find a number M between $f(a)$ and $f(b)$.

(b) Show that the conclusion to the Intermediate Value Theorem is satisfied by finding a number c in (a, b) such that $f(c) = M$.

11. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

(a) Find the slope of the line joining the points $(0, 1)$ and $(0 + h, f(0 + h)) = (h, f(h))$, where $h \neq 0$. Then find the limit as h approaches 0 to get $f'(0)$.

(b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0. In other words, find $f'(x)$.

(c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

(d) Use the information given and that found in (c) to find an equation in slope-intercept form for the line tangent to the graph of f at the point $(0, 1)$.

12. (12 points) Let

$$f(x) = \begin{cases} -1 & \text{if } x \leq 0 \\ 1 & \text{if } 0 < x < 2 \\ 3 & \text{if } 2 \leq x \end{cases}$$

and let $g(x) = 2x - 1$.

(a) Build $g \circ f$.

(b) Build $f \circ g$.