Math 1120   **Calculus, section 4**   **Test 1**

October 4, 2017   Name _____

The problems count as marked. The total number of points available is 169. The list below is an amalgamation of the four tests in the two sections, so the total number of points is greater than 169. Throughout this test, **show your work.**

1. (8 points) Find the exact value of the expression $|\pi - 7| + |2\pi - 10| + |3\pi - 8|$. Express your answer in the form $a\pi + b$ where a and b are integers.

2. (8 points) Find an equation for a line perpendicular to the line $5x - 2y = 7$ and which goes through the point $(-3, 9)$. Express your answer in slope-intercept form.

3. (12 points) How many points (x, y) in the plane satisfy both $x^2 + y^2 = 25$ and $x^2 - 10x + y^2 - 24y = -105$? The correct answer is worth 2 points, the correct explanation is worth 10 points.

4. (12 points) Suppose f, g and h are polynomials of degrees $7, 8$ and 9 respectively. What is the degree of the product $(f \circ g) \cdot (f + g + h)$, where $\circ$ means composition?

5. (30 points) Evaluate each of the limits indicated below.

    (a) $\displaystyle \lim_{x \to 2} \frac{(2x)^3 - 64}{x^2 - 4}$

    (b) $\displaystyle \lim_{h \to 0} \frac{\sqrt{25 + 2h} - 5}{h}$

    (c) $\displaystyle \lim_{h \to 0} \frac{(2 + h)^3 - 8}{h}$.

    (d) $\displaystyle \lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 + x - 2}$

    (e) $\displaystyle \lim_{x \to 3} \frac{\frac{4}{x} - \frac{4}{3}}{x - 3}$

    (f) $\displaystyle \lim_{x \to \infty} \frac{3x^4 - 6}{(11 - 3x^2)^2}$

    (g) $\displaystyle \lim_{x \to -\infty} \frac{\sqrt{36x^2 - 3}}{9x - 11}$

6. (21 points) Let

$$f(x) = \begin{cases} 2x + 3 & \text{if } -1 < x \le 0 \\ |x - 3| & \text{if } 0 < x < 4 \\ 2 & \text{if } x = 4 \\ 5 - x & \text{if } 4 < x \le 6 \end{cases},$$

(a) What is the domain of f? Express your answer in interval notation.

(b) What is $\lim\limits_{x \to 0^-} f(x)$?

(c) What is $\lim\limits_{x \to 0^+} f(x)$?

(d) Is f continuous at $x = 0$? Discuss why or why not.

(e) What is $\lim\limits_{x \to 4^-} f(x)$?

(f) What is $\lim\limits_{x \to 4^+} f(x)$?

(g) Is f continuous at $x = 4$? Discuss why or why not.

7. (10 points) Find all the x-intercepts of the function

$$g(x) = (2x^2 - 1)^2(3x + 1) - (2x^2 - 1)(3x + 1).$$

8. (20 points)

   (a) Find all solutions to $||x - 3| - 8| = 5$.

   (b) Find the domain of the function $f(x) = \sqrt{||x - 3| - 8| - 5}$ and write your answer in interval form.

9. (30 points) Let $g(x) = \sqrt{\frac{(2x-15)(3x+17)}{x^2+x-6}}$. The sequence of steps below will enable you to find the (implied) domain of g. Let $r(x) = (g(x))^2 = \frac{(2x-15)(3x+17)}{x^2+x-6}$.

(a) Find the zeros of r. That is, find all x for which $r(x) = 0$.

(b) Find the value(s) of x for which r is undefined.

(c) Write as a union of intervals the set of real numbers that result by removing the values of x found in the first two parts.

(d) For each of the intervals in part (c), select a test point in the interval, and compute the sign (plus or minus) of r at that test point.

(e) Express the domain of $g(x)$ as a union of intervals. Be sure to include or exclude the endpoints as appropriate.

10. (25 points) Let $f(x) = \sqrt{3x - 2}$. Notice that $f(6) = \sqrt{18 - 2} = 4$.

   (a) Find the slope of the line joining the points $(6, 4)$ and $(6 + h, f(6 + h))$, where $h \neq 0$. Note that $(6 + h, f(6 + h))$ is a point on the graph of f.

   (b) Compute $f(a + h)$, $f(a)$, and finally $\frac{f(a+h) - f(a)}{h}$.

   (c) Finally compute the limit as h approaches 0 to find $f'(a)$.

   (d) Replace the a with 6 to find $f'(6)$.

   (e) Use the information given and that found in (d) to find an equation for the line tangent to the graph of f at the point $(6, 4)$.

11. (20 points) Let $f(x) = x^2 - 2x$. Note that $f(3) = 3$

   (a) Find the slope of the line joining the points $(3, 3)$ and $(3 + h, f(3 + h))$, where $h \neq 0$. Note that $(3 + h, f(3 + h))$ is a point on the graph of f.

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   (c) Replace the x with 3 in your answer to (b) to find $f'(3)$.

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(3, 3)$.