September 14, 1999 **Name** _____

The first 16 problems count 5 points each and the final 3 count 15 points each.

1. Fill in the three character code you received via email in the box ☐

   Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

2. What is the y-intercept of the line passing through the points $(4, 7)$ and $(8, 2)$?

   (A) 9 **(B)** 10 **(C)** 11 **(D)** $\boxed{12}$ **(E)** 13

3. What is the exact value of $|3\pi - 10| - |2\pi - 6|$?

   (A) $4 - \pi$ **(B)** $\pi - 4$ **(C)** 0.858 **(D)** $\boxed{16 - 5\pi}$ **(E)** $5\pi + 16$

4. How many different real numbers x satisfy

   $$(x^2 + 1)(x^2 - 4)(x + 1)^2(x - 1)^2 = 0?$$

   (A) none **(B)** 2 **(C)** 3 **(D)** $\boxed{4}$ **(E)** 6

5. The domain of the function f is the interval $[2, 10]$. The function g is defined by $g(x) = f(2x)$. Which of the following does not belong to the domain of g?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** $\boxed{6}$

6. Consider the function f defined by:

   $$f(x) = \begin{cases} 2x^2 - 10 & \text{if } x < 0 \\ 5x + 3 & \text{if } x \geq 0 \end{cases}$$

   Find the slope of the line which goes through the points $(-2, f(-2))$ and $(3, f(3))$.

   (A) -4 **(B)** -2 **(C)** 2 **(D)** $\boxed{4}$ **(E)** 5

7. The expression $(x^2+2)^2[5(x+2)^3-3](2x^3)$ is a polynomial that is neither in factored form or in normal polynomial form. What is its degree?

   (A) 4 **(B)** 8 **(C)** 9 **(D)** $\boxed{10}$ **(E)** 11

8. The following points lie on the same line: $(2b,-7),(b+3,8),(b,-1)$. What is the value of b?

   (A) 3 **(B)** 2/3 **(C)** -7 **(D)** 4/3 **(E)** $\boxed{-2}$

9. The two lines $y=2kx-3$ and $y+4x=5$ are perpendicular. What is the value of the constant k?

   (A) -4 **(B)** -2 **(C)** 1/2 **(D)** 1/4 **(E)** $\boxed{1/8}$

10. Find the **sum** of the two roots of $10x^2+31x-14=0$.

    (A) -3 **(B)** $-7/5$ **(C)** 5/7 **(D)** $\boxed{-31/10}$ **(E)** 31/23

11. Let $f(x)=x^2+1$. Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$.

    (A) $h-2$ **(B)** $2x-2h+h^2$ **(C)** $\boxed{2x+h}$

    (D) $2x+h+2$ **(E)** x^2+2h+2

12. A line L is parallel to the line whose equation is $2x+3y=6$ and passes through the point $(4,-7)$. What is the slope of L?

    (A) -7 **(B)** $\boxed{-2/3}$ **(C)** 2 **(D)** 3/2 **(E)** 3

13. $(3^{-1} - 6^{-1})^{-1} =$

(A) -6 **(B)** -3 **(C)** $\frac{1}{6}$ **(D)** 3 **(E)** $\boxed{6}$

The next two problems involve the functions f and g defined below. Suppose the functions f and g are given completely by the table of values shown.

x	$f(x)$
0	2
1	7
2	5
3	1
4	3
5	6
6	0
7	4

x	$g(x)$
0	5
1	7
2	4
3	2
4	6
5	3
6	1
7	0

14. What is $f(g(4) - g(2))$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** $\boxed{5}$ **(E)** 6

15. What is $f(g(4)) - f(g(2))$?

(A) $\boxed{-3}$ **(B)** -2 **(C)** 0 **(D)** 3 **(E)** 5

16. Consider the function f defined by:

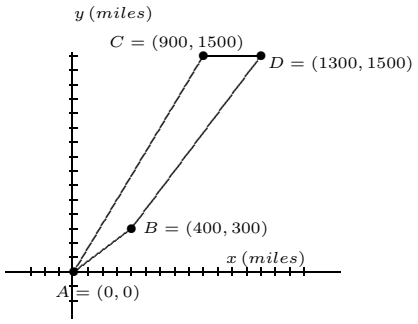
$$f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

The limit as x approaches 1

(A) $\boxed{\text{is } 1}$ **(B)** is 2 **(C)** is 3 **(D)** is 4 **(E)** does not exist

On all the following questions, **show your work.**

17. Towns A, B, C, and D are located as shown in the following figure. Two highways link town A to town D. Route 1 runs from town A to town D via town B. Route 2 runs from town A to town D via town C. If a salesman wishes to drive from town A to town D. Which of the routes provides the smallest travel time assuming that he can drive at the same speed on both routes. Explain your answer.



The distance from A to D through C is $\sqrt{900^2 + 1500^2} + 400 \approx 1749.2 + 400 = 2149.2$ whereas the trip from A to D via B has length $\sqrt{300^2 + 400^2} + \sqrt{900^2 + 1200^2} = 500 + 1500 = 2000$, so the route through B is shorter and therefore faster.

17B. The vertices of a triangle are located at $(0, 0), (2, 6)$, and $(6, 2)$. Is it an equilateral triangle. That is, do all all the sides have the same length? Give reasons and show all your work.

The distances involved are

$$D((0,0),(2,6)) = \sqrt{2^2 + 6^2} = \sqrt{40} \approx 6.32,$$

$$D((0,0),(6,2)) = \sqrt{2^2 + 6^2} = \sqrt{40} \approx 6.32, \text{ and}$$

$$D((2,6),(6,2)) = \sqrt{4^2 + 4^2} = \sqrt{32} \approx 5.65,$$

so the triangle does not have three congruent legs. Therefore it is not equilateral.

18. Using data compiled by the Admissions office at Faber University, college administrators estimate that 55% of the students who are offered admission to the freshman class actually enroll.

    (a) Find an equation that expresses the relationship between the number of students who actually enroll (y), and the number who are admitted (x).

    An equation relating x and y is $y = .55x$.

    (b) If the desired freshman class size is 1100 students, how many students should be admitted?

    We must solve the equation $1100 = .55x$ for the unknown x. Thus $x = 1100/.55 = 110000/55 = 2000$.

19. The two equations given are supply and demand equations, where x represents the number of units (in thousands) and p represents the price in dollars. Find the equilibrium price to the nearest penny and the equilibrium quantity.

$$11p + 3x - 66 = 0 \text{ and } 2p^2 + p - x = 12$$

Use the quadratic formula to solve $2p^2 + p - (66 - 11p)/3 = 12$. You get $p = 3.233$ and $x = 10.14$.