February 25, 2019 **Name** ______________________________

The problems count as marked. The total number of points available is 155. Throughout this test, **show your work.**

1. (6 points) Find an equation in slope-intercept form for a line perpendicular to the line $3x - 6y = 7$ and which goes through the point $(-3, 5)$.

   Solution: The given line has slope $1/2$ so the one perpendicular has slope -2. Hence $y - 5 = (-2)(x + 3)$. Thus $y = -2x - 1$.

2. (20 points) The equations $x^2 + 2x + y^2 = 15$ and $x^2 - 10x + y^2 - 16y = -53$ are both circles.

   (a) (8 points) Use the 'complete the square' idea to find the centers and radii of the circles.

   Solution: The first is $(x + 1)^2 + y^2 = 4^2$ and the second is $(x - 5)^2 + (y - 8)^2 = 6^2$ so the centers are $(-1, 0)$ and $(5, 8)$ and the radii are 4 and 6 respectively.

   (b) Find the distance between the centers.

   Solution: The distance between the centers is $\sqrt{6^2 + 8^2} = 10$.

   (c) Find the midpoint of the line segment joining the centers.

   Solution: The midpoint of the segment is $\frac{1}{2}(-1, 0) + \frac{1}{2}(5, 8) = (2, 4)$.

   (d) Find the slope of the line joining the centers.

   Solution: The slope is $8/6 = 4/3$.

   (e) Do the circles have one, two, or no points in common? Write a complete sentence to justify your answer.

   Solution: The circles have one point in common because the sum of the radii is exactly the distance between the centers. That point is $(1.4, 3.2)$.

3. (42 points) Evaluate each of the limits (and function values) indicated below.

   (a) $\lim_{x\to 2} \dfrac{(x+1)^2-9}{x-2}$

   Solution: Factor and eliminate the $x-2$ from numerator and denominator to get

   $$\lim_{x\to 2} x+4 = 6$$

   (b) $\lim_{x\to 2} \dfrac{2-x}{\frac{1}{2x}-\frac{1}{4}}$

   Solution: The limit of both the numerator and the denominator is 0, so we must do the fractional arithmetic. The limit becomes

   $$\begin{aligned} \lim_{x\to 2} \frac{2-x}{\frac{2-x}{4x}} &= \\ \lim_{x\to 2} 4x &= 8. \end{aligned}$$

   (c) $\lim_{x\to 6} \dfrac{\sqrt{2x-3}-3}{x-6}$

   Solution: Rationalize the numerator to get $\lim_{x\to 6} \frac{(\sqrt{2x-3}-3)(\sqrt{2x-3}+3)}{(x-6)(\sqrt{2x-3}+3)} = \lim_{x\to 6} \frac{2x-3-9}{(x-6)(\sqrt{2x-3}+3)} = 2/6 = 1/3.$

   (d) $\lim_{x\to -1} \dfrac{x^3+6x^2+11x+6}{x^3-4x^2+x+6}$

   Solution: Factor both parts to get $\lim_{x\to -1} \dfrac{(x+1)(x^2+5x+6)}{(x+1)(x^2-5x+6} = 2/12 = 1/6.$

   (e) $\lim_{x\to 1} \dfrac{x^2-1}{x^3-1}$

   Solution: Factor both numerator and denominator to get $\lim_{x\to 1} \frac{x^2-1}{x^3-1} = \lim_{x\to 1} \frac{(x-1)(x+1}{(x-1)(x^2+x+1)} = \lim_{x\to 1} \frac{x+1}{x^2+x+1} = 2/3$

   (f) $\lim_{h\to\infty} \dfrac{(2x^2-5)(3x+1)}{4x^3+x^2-17}.$

   Solution: The degrees are the same so we get $6/4 = 3/2$.

4. (18 points) Let

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x-1 & \text{if } 0 \le x < 2 \\ -1 & \text{if } x = 2 \\ 1 & \text{if } 2 < x \le 7 \end{cases}$$

Find the value, if it exists, of each item below. Use DNE when the limit does not exist.

(a) What is the domain of f?
Solution: $(-\infty, 7]$.

(b) $\lim_{x\to 0^-} f(x)$
Solution: 0

(c) $\lim_{x\to 0^+} f(x)$
Solution: -1

(d) $\lim_{x\to 0} f(x)$
Solution: DNE because the left limit and right limit are different.

(e) $f(0)$
Solution: -1

(f) $\lim_{x\to 2^-} f(x)$
Solution: 1

(g) $\lim_{x\to 2^+} f(x)$
Solution: 1, because both the left limit and the right limit are 1.

(h) $\lim_{x\to 2} f(x)$
Solution: 1

(i) $f(2)$
Solution: -1

5. (10 points) Find all the x-intercepts of the function

$$g(x) = (2x^2 - 1)^2(3x + 1) - (2x^2 - 1)(3x + 1)^2.$$

Solution: Factor out the common terms to get $g(x) = (2x^2-1)(3x+1)[(2x^2-1) - (3x+1) = (2x^2-1)(3x+1)(2x^2-3x-2)$. Setting each factor equal to zero, we find the zeros are $x = -\sqrt{2}/2, x = \sqrt{2}/2,\ x = -1/3, x = 1$ and $x = -3$.

6. (15 points)

   (a) Find all solutions of the inequality $\mid 2x - 7 \mid\leq 5$ and write your solution in interval notation.

   Solution: First solve the equation $\mid 2x-7 \mid\leq 5$, which has two solutions: $2x - 7 = 5$ yields $x = 6$ and $2x - 7 = -5$ yields $x = 1$. Now consider the three intervals determined by these two points: $(-\infty, 1), (1, 6), (6, \infty)$. Select a test point from each of these intervals. I've picked $0, 3$, and 7. Trying each of these, we see that $\mid 2\cdot 0-7 \mid= 7 \leq 5$, NO; $\mid 2\cdot 3-7 \mid= 1 \leq 5$, YES; $\mid 2 \cdot 7 - 7 \mid= 7 \leq 5$, NO; So only the interval $(1, 6)$ works. Check the endpoints and see that they both work also. So our answer is $[1, 6]$.

   (b) Find the (implied) domain of

   $$f(x) = \sqrt{\mid 2x - 7 \mid -3},$$

   and write your answer in interval notation.

   Solution: We need to find out where $\mid 2x - 7 \mid -3$ is zero or positive. First solve the equation $\mid 2x-7 \mid -3 = 0$ to get $x = 5$ and $x = 2$. Then use the test interval technique to find the sign chart for $g(x) =\mid 2x - 7 \mid -3$. You see that $g(x) > 0$ on both $(-\infty, 2)$ and $(5, \infty)$. Then notice that the endpoints need to be included. So the answer is $(-\infty, 2] \cup [5, \infty)$.

7. (24 points) Compute the following derivatives.

   (a) Let $f(x) = \frac{x^2-2x}{3x-x^2}$. Find $\frac{d}{dx}f(x)$.

   Solution: Use the quotient rule to get $f'(x) = \frac{d}{dx}f(x) = \frac{(2x-2)(3x-x^2)-(3-2x)(x^2-2x)}{(3x-x^2)^2}$. This can be simplified to $-\frac{x^2}{(3x-x^2)^2}$ which can be simplified still further: $-\frac{1}{(x-3)^2}$.

   (b) Let $g(x) = \sqrt{x^3+2x+4}$. What is $g'(x)$?

   Solution: $g'(x) = 1/2(x^3+2x+4)^{-1/2} \cdot (3x^2+2) = \frac{3x^2+2}{2\sqrt{x^3+2x+4}}$.

   (c) Find $\frac{d}{dx}((3x+1)^2 \cdot (4x^2-1))$
   Solution: Use the product rule to get $\frac{d}{dx}((3x+1)^2 \cdot (4x^2-1)) = 2(3x+1)^1 \cdot 3 \cdot (4x^2-1) + 8x(3x+1)^2$, which can be simplified but not significantly.

   (d) Let $f(x) = (2x^2+1)^4$. Find $f'(x)$.

   Solution: Note that, by the chain rule, $f'(x) = 4(2x^2+1)^3 \cdot 4x = 16x(2x^2+1)^3$.

8. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

   (a) Find the slope of the line joining the points $(0, 1)$ and $(0+h, f(0+h)) = (h, f(h))$, where $h \neq 0$.

   Solution: $\frac{f(h)-1}{h-0}$, which can be massaged to give $\frac{1}{h-1}$.

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \\ &= \lim_{h\to 0} \frac{\frac{x+1-(x+h+1)}{(x+1)(x+h+1)}}{h} \\ &= \lim_{h\to 0} \frac{\frac{-h}{(x+1)(x+h+1)}}{h} \\ &= -\frac{1}{(x+1)^2}. \end{aligned}$$

   (c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

   Solution: $f'(0) = -1$

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(0, 1)$.

   Solution: The line is $y - 1 = -1(x - 0)$, or $y = -x + 1$.