February 14, 2003 **Name** ______________________________

The first 7 problems count 7 points each and the final 4 count as marked. The total number of points available is 129.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Which of the following numbers belong to the (implied) domain of
$$f(x) = \frac{\sqrt{x-2}}{x-3}?$$
Circle all those that apply.

   (A) -2 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

   Solution: The domain includes all real numbers greater than or equal to 2 except 3, which makes the denominator zero. Thus, 2, 4, and 5 all belong to the domain.

2. What is the y-intercept of the line defined by $\frac{x}{6} + \frac{y}{3} = 2$?

   (A) -2 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 16

   Solution: The y-intercept is the point on the line for which $x = 0$. Solving for y gives $y = 6$.

3. Let $f(x) = 2x + 4$ and $g(x) = 3x - 9$. What is the value of $g(f(g(3)))$?

   (A) -18 **(B)** -3 **(C)** 3 **(D)** 9 **(E)** 18

   Solution: $g(f(g(3))) = g(f(0)) = g(4) = 3$.

4. Let $f(x) = x^2 + 1$. Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$.

   (A) $h - 2$ **(B)** $2x - 2h + h^2$ **(C)** $2x + h$

   (D) $2x + h + 2$ **(E)** $x^2 + 2h + 2$

   Solution: Simplify $\frac{(x+h)^2+1-(x^2+1)}{h}$ to get $\frac{x^2+2xh+h^2+1-x^2-1)}{h} = \frac{2xh+h^2}{h}$, whereupon, the h can be factored from the numerator and cancelled with the h in the denominator to yield $2x + h$.

5. Referring to the function $h(x)$ defined in problem 9, what is the slope of the secant line joining the points $(-2, h(-2))$ and $(4, h(4))$?

   (A) -1 **(B)** $-1/2$ **(C)** 0 **(D)** $1/2$ **(E)** 1

   Solution: A. The slope is $m = \frac{3-0}{-2-4} = -\frac{1}{2}$.

Suppose the functions f and g are given completely by the table of values shown.

x	$f(x)$	x	$g(x)$
0	2	0	5
1	7	1	7
2	5	2	4
3	1	3	2
4	3	4	6
5	6	5	3
6	0	6	1
7	4	7	0

6. Solve the equation $f \circ g(x) = 6$?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 5 **(E)** 6

Solution: Since $f(5) = 6$, it must be the case that $g(x) = 5$. This is true only when $x = 0$.

7. Compute $(g \circ f)(2 + f(2))$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Solution: Note that $2 + f(2) = 7$ and $g(f(7)) = g(4) = 6$.

On all the following questions, **show your work.**

8. (10 points) The supply and demand curves are given below for digital cameras at XYZ Distributors, where x represents the number of units and p the price. Find the equilibrium quantity and price. Demand: $p = -x^2 - 2x + 100$ and Supply: $p = 8x + 25$.

Solution: Solve $-x^2 - 2x + 100 = 8x + 25$ by solving the quadratic $-x^2 - 10x + 75 = 0$ to get two solutions, $x = 5$ and $x = -15$, the later of which is extraneous. Thus $x = 5$ and $p = 65$.

9. (30 points) For each of the next questions, let h be defined as follows:

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ x & \text{if } 0 \leq x < 2 \\ 3 & \text{if } x = 2 \\ 4 - x & \text{if } 2 < x \end{cases}$$

(a) What is $\lim_{x \to -1} h(x)$?

Solution: $\lim_{x \to -1} h(x) = (-1)^2 - 1 = 0$

(b) What is $\lim_{x \to 0^-} h(x)$?

Solution: $\lim_{x \to 0^-} h(x) = \lim_{x \to 0^-} x^2 - 1 = -1$

(c) What is $\lim_{x \to 1} h(x)$?

Solution: $\lim_{x \to 1} h(x) = \lim_{x \to 0^-} x = 1$

(d) What is $\lim_{x \to 2^+} h(x)$?

Solution: $\lim_{x \to 2^+} h(x) = \lim_{x \to 2^+} 4 - x = 2$

(e) What is $\lim_{x \to 2} h(x)$?

Solution: Since $\lim_{x \to 2^-} h(x) = \lim_{x \to 2^-} x = 2$ and the limit from the right is also 2, it follows that the limit is 2.

(f) What is $\lim_{x \to 4} h(x)$?

Solution: $\lim_{x \to 4} h(x) = \lim_{x \to 4} 4 - x = 0$

10. (40 points) Compute each of the following limits.

(a) Let $f(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

$\lim_{x\to 1} f(x)$

Solution: Use the blotter test to see that $f(x)$ is close to 3 when x is close (but not equal) to 1.

(b) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

Solution: Factor the numerator and cancel out the factor $x-2$ to get $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{x+2}{1} = 4$.

(c) $\lim_{x\to 1} \frac{x-1}{x^3-1}$

Solution: Factor the denominator and cancel out the factor $x-1$ to get $\lim_{x\to 1} \frac{1}{x^2+x+1} = 1/3$.

(d) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

Solution: Rationalize the numerator to get $\frac{\sqrt{x}-3}{x-9} = \frac{x-9}{(x-9)(\sqrt{x}+3)}$ which has limit $1/6$ as x approaches 9.

(e) $\lim_{x\to 1} \frac{\frac{1}{2x}-\frac{1}{2}}{x-1}$

Solution: Do the fraction arithmetic to get $\frac{\frac{1}{2x}-\frac{1}{2}}{x-1} = \frac{\frac{1-x}{2x}}{\frac{x-1}{1}} = -\frac{1}{2x}$ which has limit $-1/2$ as x approaches 1.

(f) $\lim_{x\to 2} \frac{x^2-2x}{x^2+x-6}$

Solution: Factor and eliminate the common factor $x-2$, then set $x=2$ to get $2/(2+3) = 2/5$.

(g) $\lim_{x\to 2} 2x^3\sqrt{x^2+12}$

Solution: Just replace all the x's with the number 2 to get $2\cdot 2^3\sqrt{4+12} = 16\cdot 4 = 64$.

(h) $\lim_{x\to\infty} \frac{2x^2}{1+x^2}$

Solution: We are looking for the horizontal asymptote, which by the asymptote theorem is just $2/1 = 2$.