October 2, 2019 **Name** ________________________

The problems count as marked. The total number of points available is 171. Throughout this test, **show your work.** This is an amalgamation of the tests from sections 4 and 5.

1. (10 points) Find an equation for a line parallel to the line $2y + 3x = 12$ which passes through the point $(3, 5)$.

2. (10 points) Write the set of points that satisfy $||2x - 15| - 3| \leq 2$ using interval notation.

3. (25 points) The set C of points satisfying $x^2 - 24x + y^2 - 10y = -160$ is a circle.

   (a) Find the center and the radius of the circle C.

   (b) There are two circles centered at $(0, 0)$ which are tangent to C. Find an equation for one of them.

   (c) Find the point on the line $y = x$ that is closest to C.

4. (36 points) Evaluate each of the limits indicated below.

   (a) $\displaystyle\lim_{x\to 3} \frac{x^3-8}{x-2}$

   (b) $\displaystyle\lim_{x\to -2} \frac{x^3-3x^2-24x-28}{x^2-4}$

   (c) $\displaystyle\lim_{x\to -2} \frac{x^3-3x^2-24x-28}{x^3+2x^2-4x+8}$

   (d) $\displaystyle\lim_{x\to 1} \frac{x^3-5x^2+7x-3}{x^3-3x+2}$ Hint: think about why this is a zero over zero problem.

   (e) $\displaystyle\lim_{x\to 2} \frac{\frac{1}{3x}-\frac{1}{6}}{x-2}$

   (f) $\displaystyle\lim_{x\to 3} \frac{\sqrt{22-6x}-2}{x-3}$

   (g) $\displaystyle\lim_{x\to -\infty} \frac{(2-x)(10+6x)^2}{(3-5x)(8+8x)^2}$

   (h) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{4x^6-6}}{x^2+6}$

   (i) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{4x^6-6}}{x^3+6}$

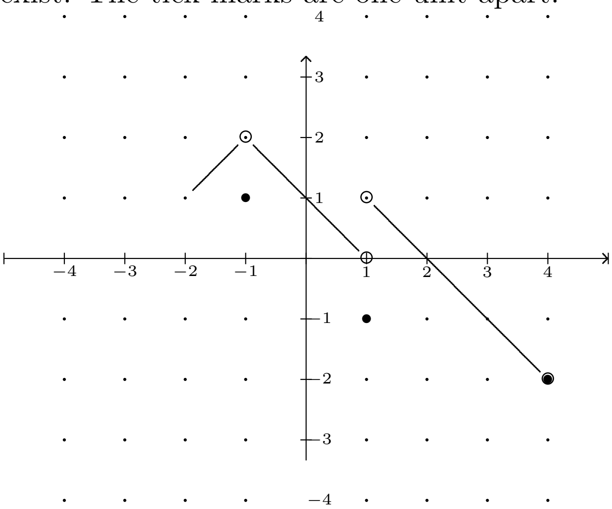
5. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{(x+4)(2x-3)(3x-17)}}{x-6}.$$

Express your answer as a union of intervals. That is, use interval notation.

6. (12 points) Let $H(x) = (x+1)(x^2-9) - (x-3)(3x+5)$. Find the zeros of the function.

7. (18 points) Consider the function F whose graph is given below. Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist. The tick marks are one unit apart.



(a) $\lim_{x \to -1^-} F(x) =$

(b) $\lim_{x \to -1^+} F(x) =$

(c) $\lim_{x \to -1} F(x) =$

(d) $F(-1) =$

(e) $\lim_{x \to 1^-} F(x) =$

(f) $\lim_{x \to 1^+} F(x) =$

(g) $\lim_{x \to 1} F(x) =$

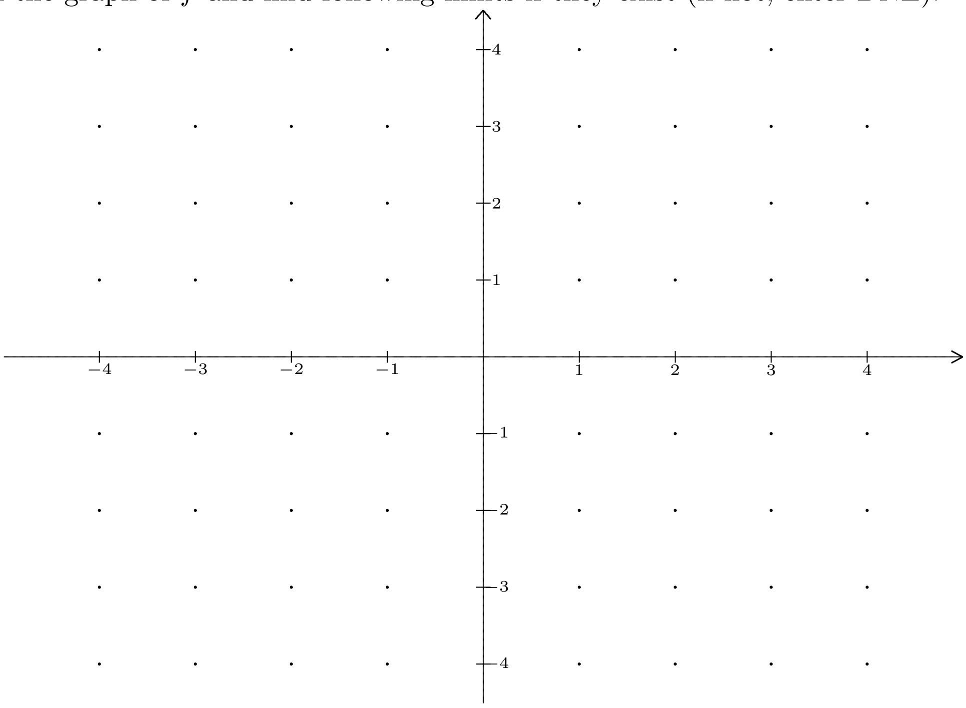
(h) $\lim_{x \to 3} F(x) =$

(i) $F(3) =$

8. (18 points)

$$f(x) = \begin{cases} 3 & \text{if } 2 < x \leq 4 \\ 4 & \text{if } x = 2 \\ -x+1 & \text{if } 0 \leq x < 2 \\ x+1 & \text{if } -4 \leq x < 0 \end{cases}$$

Sketch the graph of f and find following limits if they exist (if not, enter DNE).



(a) Express the domain of f in interval notation.

(b) $\lim_{x \to 2^-} f(x)$

(c) $\lim_{x \to 2^+} f(x)$

(d) $\lim_{x \to 2} f(x)$

(e) $\lim_{x \to 0^-} f(x)$

(f) $\lim_{x \to 0} f(x)$

9. (12 points) Let $f(x) = (2x^2 - 3)^3(5x - 1) + 17x^5$, let $g(x) = (3x - 4)(2x^3)^2 - 2x^4$.

   (a) What is the degree of the polynomial $f + g$?

   (b) What is the degree of the polynomial $f \cdot g$?

   (c) Estimate within one tenth of a unit the value of $f(10000)/g(10000)$.

   (d) Compute $\lim_{x\to\infty} \frac{f(x)}{g(x)}$.

10. (15 points) Recall that the Intermediate Value Theorem guarantees that for any function f continuous over the interval $[a, b]$ and for any number M between $f(a)$ and $f(b)$, there exists a number c such that $f(c) = M$. The function $f(x) = \frac{1}{1+\frac{1}{x}}$ is continuous for all $x > 0$. Let $a = 1$.

    (a) Pick a number $b > 1$ (any choice is right), and then find a number M between $f(a)$ and $f(b)$.

    (b) Show that the conclusion to the Intermediate Value Theorem is satisfied by finding a number c in (a, b) such that $f(c) = M$.

11. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

    (a) Find the slope of the line joining the points $(0, 1)$ and $(0 + h, f(0 + h)) = (h, f(h))$, where $h \neq 0$. Then find the limit as h approaches 0 to get $f'(0)$.

    (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0. In other words, find $f'(x)$.

    (c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

    (d) Use the information given and that found in (c) to find an equation in slope-intercept form for the line tangent to the graph of f at the point $(0, 1)$.

12. (12 points) Let

$$f(x) = \begin{cases} -1 & \text{if } x \leq 0 \\ 1 & \text{if } 0 < x < 2 \\ 3 & \text{if } 2 \leq x \end{cases}$$

and let $g(x) = 2x - 1$.

(a) Build $g \circ f$.

(b) Build $f \circ g$.