Math 1120 **Calculus** **Test 1**

February 13, 2004 **Name** ____________________________

The first 12 problems count 7 points each and the final one counts 40 points. The total number of points available is 124. Throughout this test, **show your work.**

1. What is the degree of the polynomial $p(x) = (x^2 - 1)^3(x^3 + 7)$?

   Solution: $(x^2 - 1)^3$ has degree 6 and $(x^3 + 7)$ has degree 3, so $p(x)$ has degree $6 + 3 = 9$.

2. Let P denote the midpoint of the line segment joining $(2, 3)$ and $(8, 11)$. What is the distance from P to the point $(-2, 3)$?

   Solution: $P = (\frac{2+8}{2}, \frac{3+11}{2}) = (5, 7)$ so the distance is $d = \sqrt{(5+2)^2 + (7-3)^2} = \sqrt{49 + 16} = \sqrt{65}$.

3. Find the (implied) domain of

$$f(x) = \frac{\sqrt{x-4}}{x-7},$$

   and write your answer in interval notation.

   Solution: The domain D includes all real numbers greater than or equal to 4 except 7, which must be eliminated because it makes the denominator zero. Thus, $D = [4, 7) \cup (7, \infty)$.

4. Find all the x-intercepts of the function

$$t(x) = (2x-1)^3(x+1)^2 - (2x-1)^2(x+1)^3.$$

   Solution: Factor the common stuff out to get $(2x-1)^2(x+1)^2[2x-1-(x+1)]$. Setting each of the three factors to zero yields $x = 1/2, x = -1$, and $x = 2$.

5. The line tangent to the graph of $y = e^{2x}$ at the point $(0, 1)$ has slope 2. What is the x-intercept of the line?

   Solution: The line is $y - 1 = 2(x - 0)$ so the x-intercept is $-1/2$.

6. Consider the rational function $k(x) = \frac{(2x+1)^2(x+5)}{3x^3-5x^2}$. Estimate the value $k(1000)$. Does k have a horizontal asymptote? Discuss.

   Solution: Yes, $k(1000) \cong 1.34$ and the horizontal asymptote is the horizontal line whose value is the ratio of the coefficients of x^3 in the numerator and denominator, $y = 4/3$

7. Find an equation for a line perpendicular to the line $3x - 4y = 7$ and which goes through the point $(-2, -3)$.

   Solution: The given line has slope $3/4$ so the one perpendicular has slope $-4/3$. Hence $y + 3 = (-4/3)(x + 2)$. Thus $y = -4x/3 - 17/3$.

8. Let $k(x) = x^2 - x$. Evaluate and simplify $\frac{k(x+h)-k(x)}{h}$. Then find the limit of the expression as h approaches 0.

   Solution:

$$
\begin{aligned}
k'(x) &= \lim_{h \to 0} \frac{k(x+h) - k(x)}{h} \\
&= \lim_{h \to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2 \cdot xh + h^2 - x - h - x^2 + x}{h} \\
&= \lim_{h \to 0} \frac{+2 \cdot xh + h^2 - h}{h} \\
&= \lim_{h \to 0} \frac{h(2x + h - 1)}{h} = 2x - 1.
\end{aligned}
$$

9. Suppose the functions f and g are given completely by the table of values shown.

x	$f(x)$	x	$g(x)$
0	2	0	5
1	7	1	7
2	5	2	4
3	1	3	2
4	3	4	6
5	6	5	3
6	0	6	1
7	4	7	0

10. What is $f \circ g \circ f(2)$?

    Solution: Note that $f(2) = 5, g(5) = 3$, and $f(3) = 1$, so $f \circ g \circ f(2) = 1$.

11. Solve $(f \circ g)(x) = 7$?

    Solution: Note that $g(x)$ must be 1, so $x = 6$.

12. (10 points) The supply and demand curves are given below for digital cameras at XYZ Distributors, where x represents the number of units and p the price. Find the equilibrium quantity and price. Demand: $p = -x^2 - 2x + 100$ and Supply: $p = 10x + 55$.

    Solution: Solve $-x^2 - 2x + 100 = 10x + 55$ by solving the quadratic $-x^2 - 12x + 45 = 0$ to get two solutions, $x = 3$ and $x = -15$, the later of which is extraneous. Thus $x = 3$ and $p = 85$.

13. (40 points) Compute each of the following limits.

(a) Let $f(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

$\lim_{x\to 1} f(x)$

Solution: 3. Use the blotter test to see that $f(x)$ is close to 3 when x is close (but not equal) to 1.

(b) $\lim_{x\to 0} \dfrac{x^2-2x}{x}$

Solution: Factor the numerator and cancel out the factor x to get $\lim_{x\to 0} = -2$.

(c) $\lim_{x\to 3} \dfrac{x^2-3x}{x^2+x-12}$

Solution: Factor and eliminate the common factor $x-3$, then set $x=3$ to get $3/(3+4) = 3/7$.

(d) $\lim_{x\to 2} |x^2 - \sqrt{16x-7}|$

Solution: Just replace all the x's with the number 2 to get $|2^2 - \sqrt{16\cdot 2 - 7}| = |4-5| = 1$.

(e) $\lim_{x\to 1} \dfrac{x^2-1}{x^3-1}$

Solution: Factor the denominator and cancel out the factor $x-1$ to get $\lim_{x\to 1} \dfrac{x+1}{x^2+x+1} = 2/3$.

(f) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$

Solution: Rationalize the denominator to get $\frac{x-9}{\sqrt{x}-3} = \frac{(x-9)(\sqrt{x}+3)}{x-9}$ which has limit 6 as x approaches 9.

(g) $\lim_{x\to 1} \dfrac{\frac{1}{3x} - \frac{1}{3}}{x-1}$

Solution: Do the fraction arithmetic to get $\frac{\frac{1}{3x}-\frac{1}{3}}{x-1} = \frac{\frac{1-x}{3x}}{\frac{x-1}{1}} = -\frac{1}{3x}$ which has limit $-1/3$ as x approaches 1.

(h) $\lim_{x\to\infty} \dfrac{2x^2}{(1-x)^2}$

Solution: We are looking for the horizontal asymptote, which by the asymptote theorem is just $2/1 = 2$.