February 13, 2007 **Name** _____

The problems count as marked. The total number of points available is 135. Throughout this test, **show your work.**

1. (40 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to 0} \frac{x^4-x^2}{x^2}$

   (b) $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-1}$

   (c) $\lim_{x\to 5} \frac{x-5}{x^2-3x-10}$

   (d) $\lim_{x\to\infty} \frac{\sqrt{9x^2-3}}{11-5x}$

   For problems (e) through (j), let

$$f(x) = \begin{cases} 7-x & \text{if } x > 2 \\ 10 & \text{if } x = 2 \\ 2x+1 & \text{if } 0 \le x < 2 \\ -1 & \text{if } x < 0 \end{cases}$$

   (e) $\lim\limits_{x\to 0^-} f(x)$

   (f) $\lim\limits_{x\to 0^+} f(x)$

   (g) $\lim\limits_{x\to 0} f(x)$

   (h) $\lim\limits_{x\to 2^-} f(x)$

   (i) $\lim\limits_{x\to 2^+} f(x)$

   (j) $\lim\limits_{x\to 2} f(x)$

2. (21 points) Consider the function whose properties are displayed.

a	-1	0	1	2	3	4
$\lim_{x\to a^-} f(x)$	DNE	1	1	4	2	3
$\lim_{x\to a^+} f(x)$	1	2	1	3	2	DNE
$f(a)$	1	2	-1	1	2	3
$\lim_{x\to a^-} g(x)$	4	1	3	3	1	0
$\lim_{x\to a^+} g(x)$	1	2	0	3	1	DNE
$g(a)$	1	-1	3	3	DNE	0

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a) $\lim_{x\to 2^+}[f(x)+g(x)]$

(b) $\lim_{x\to 2^-}[f(x)+g(x)]$

(c) $\lim_{x\to 2}[f(x)+g(x)]$

(d) $(f+g)(4)$

(e) $f\circ g\circ f(-1)$

(f) Find all points (in the table) at which f is continuous.

(g) Find all points (in the table) at which g is continuous.

3. (7 points) Compute the exact value of $|2-4\pi|+|8-2\pi|+|6-6\pi|$. No points for a decimal approximation.

4. (10 points) Find the (implied) domain of

$$f(x)=\sqrt{(x-2)(x^2-9)},$$

and write your answer in interval notation.

5. (25 points) Let $f(x) = \sqrt{3x-2}$. Notice that $f(6) = \sqrt{18-2} = 4$.

   (a) Find the slope of the line joining the points $(6, 4)$ and $(6+h, f(6+h))$, where $h \neq 0$. Note that $(6+h, f(6+h))$ is a point on the graph of f.

   (b) Compute $f(a+h)$, $f(a)$, and finally $\frac{f(a+h)-f(a)}{h}$.

   (c) Finally compute the limit as h approaches 0 to find $f'(a)$.

   (d) Replace the a with 6 to find $f'(6)$.

6. (32 points) Given three functions, $h(x) = 2x$,

$$g(x) = \begin{cases} 3x - 1 & \text{if } x > 6 \\ 4 - x & \text{if } x \leq 6 \end{cases} \quad \text{and} \quad f(x) = \begin{cases} \sqrt{x+3} & \text{if } x \geq 1 \\ x^2 & \text{if } x < 1 \end{cases}$$

Note that $f \circ g \circ h(-2) = f \circ g(h(-2)) = f \circ g(-4) = f(8) = \sqrt{11}$.

(a) Complete the following table.

x	$h(x)$	$g \circ h(x)$	$f \circ g \circ h(x)$
-2	-4	8	$\sqrt{11}$
4			
	10		
		-2	
			0

(b) Find all solutions to $f \circ g \circ h(x) = 3$.

(c) Find a symbolic representation of $g \circ h(x)$.