**September 27, 2007** **Name** _____

The problems count as marked. The total number of points available is 132. Throughout this test, **show your work.**

1. (40 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to 4} \dfrac{\frac{2}{x} - \frac{1}{2}}{x-4}$

   (b) $\lim_{x\to 16} \dfrac{\sqrt{x}-4}{x-16}$

   (c) $\lim_{x\to 0} \frac{x^3+2x^2}{x^2}$

   (d) $\lim_{x\to -2} \frac{x^3+8}{x+2}$

   (e) $\lim_{x\to \infty} \frac{11+5x}{\sqrt{9x^2-3}}$

For problems (f) through (k), let

$$f(x) = \begin{cases} 7 - x & \text{if } x < 0 \\ 10 & \text{if } x = 0 \\ (x+1)(x+7) & \text{if } 0 \leq x < 3 \\ 30 & \text{if } 3 \leq x \end{cases}$$

(f) $\lim_{x \to 0^-} f(x)$

(g) $\lim_{x \to 0^+} f(x)$

(h) $\lim_{x \to 0} f(x)$

(i) $\lim_{x \to 3^-} f(x)$

(j) $\lim_{x \to 3^+} f(x)$

(k) $\lim_{x \to 3} f(x)$

2. (10 points) When $|2 - 4\pi - 3\sqrt{2}| + |4\sqrt{2} + 8 - 2\pi| + |6 - 6\pi - \sqrt{8}|$ is expressed in the form $a + b\sqrt{2} + c\pi$, where $a, b$, and $c$ are integers, what are the values of $a, b$, and $c$? No points for a decimal approximation.

3. (21 points) Consider the function whose properties are displayed.

| $a$ | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $\lim_{x\to a^-} f(x)$ | 1 | 1 | 1 | 3 | 2 | 3 |
| $\lim_{x\to a^+} f(x)$ | 1 | 2 | 1 | 3 | 2 | 3 |
| $f(a)$ | 1 | 2 | -1 | 1 | 4 | 3 |
| $\lim_{x\to a^-} g(x)$ | 4 | 1 | 3 | 3 | 1 | 4 |
| $\lim_{x\to a^+} g(x)$ | 1 | 2 | 0 | 3 | 1 | 4 |
| $g(a)$ | 1 | -1 | 3 | DNE | DNE | 4 |

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a) $\lim_{x\to 0^+}[f(x)+g(x)]$

(b) $\lim_{x\to 0^-}[f(x)+g(x)]$

(c) $\lim_{x\to 2}[f(x)+g(x)]$

(d) $(f+g)(4)$

(e) $f\circ g\circ f(-1)$

(f) Find all points (in the table) at which $f$ is continuous.

(g) Find all points (in the table) at which $g$ is continuous.

4. (18 points) Find the (implied) domain of each of the functions given below. Write your answers in interval notation.

   (a) $f(x) = \sqrt{(x-2)(x-3)} - \sqrt{(x-5)(x-7)}$.

   (b) $g(x) = (2x^2 + 5x - 12)^{-1}$.

5. (25 points) Let $f(x) = \sqrt{2x+1}$. Notice that $f(4) = \sqrt{9} = 3$.

   (a) Find the slope of the line joining the points $(4, 3)$ and $(4+h, f(4+h))$, where $h \neq 0$. Note that $(4+h, f(4+h))$ is a point on the graph of $f$.

   (b) Compute $f(a+h)$, $f(a)$, and finally $\frac{f(a+h)-f(a)}{h}$.

   (c) Finally compute the limit as $h$ approaches 0 to find $f'(a)$.

   (d) Replace the $a$ with 4 to find $f'(4)$.

6. (32 points) Given three functions, $h(x) = 2x$,

$$g(x) = \begin{cases} x^2 + 1 & \text{if } x > 3 \\ 4 - x & \text{if } x \leq 3 \end{cases} \quad \text{and} \quad f(x) = \begin{cases} \sqrt{x+3} & \text{if } x \geq 2 \\ 2x - 1 & \text{if } x < 2 \end{cases}$$

Note that $f \circ g \circ h(-2) = f \circ g(h(-2)) = f \circ g(-4) = f(8) = \sqrt{11}$.

(a) Complete the following table.

| $x$ | $h(x)$ | $g \circ h(x)$ | $f \circ g \circ h(x)$ |
|---|---|---|---|
| -2 | -4 | 8 | $\sqrt{11}$ |
| 3/2 | | | |
| | 10 | | |
| | | 10 | |
| | | | 3 |

(b) Find all solutions to $f \circ g \circ h(x) = 3$.

(c) Find a symbolic representation of $g \circ h(x)$.