Math 1120 **Calculus, section 6** **Test 1**

February 14, 2008 **Name** ______________________________

The problems count as marked. The total number of points available is 142. Throughout this test, **show your work.**

1. (8 points) Find an equation for a line perpendicular to the line $3x - 6y = 7$ and which goes through the point $(-3, 4)$.

   Solution: The given line has slope 1/2 so the one perpendicular has slope -2. Hence $y - 4 = (-2)(x + 3)$. Thus $y = -2x - 2$.

2. (52 points) Evaluate each of the limits indicated below.

   (a) $\lim\limits_{x\to 1} \dfrac{x^2 + x - 2}{x^2 - 4x + 3}$

   Solution: Factor and eliminate the $x - 1$ from numerator and denominator to get

   $$\lim_{x\to 1} \frac{x+2}{x-3} = -3/2$$

   (b) $\lim\limits_{x\to 5} \dfrac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

   Solution: The limit of both the numerator and the denominator is 0, so we must do the fractional arithmetic. The limit becomes

   $$\lim_{x\to 5} \frac{\frac{5-x}{5x}}{x-5} = \lim_{x\to 5} \frac{-\frac{x-5}{5x}}{x-5}$$
   $$= \lim_{x\to 5} \frac{-\frac{1}{5x}}{1} = -1/25.$$

   (c) $\lim\limits_{x\to -\infty} \dfrac{|\ 16x - 3\ |}{11 - 5x}$

   Solution: Divide both numerator and denominator by x to get $\lim_{x\to-\infty} \frac{3/x - 16x/x}{11/x - 5x/x} =$ 16/5 because the degree of the denominator is essentially the same as that of the numerator.

   (d) $\lim\limits_{x\to\infty} \dfrac{6x^2 - 3}{11 - 5x^3}$

   Solution: $\lim_{x\to\infty} \frac{6x^2-3}{11-5x^3} = \lim_{x\to\infty} \frac{6x^2/x^3 - 3/x^3}{11/x^3 - 5x^3/x^3} = 0/(-5) = 0$ because the degree of the denominator is the larger than that of the numerator.

(e) $\lim_{x \to -1} \dfrac{x^3 + 1}{x^2 - 1}$

Solution: Factor both numerator and denominator to get $\lim_{x\to-1} \frac{x^3-1}{x^2-1} = \lim_{x\to-1} \frac{(x+1)(x^2-x+1)}{(x-1)(x+1)} = \lim_{x\to-1} \frac{x^2-x+1}{x-1} = -3/2$

(f) $\lim_{h \to 0} \dfrac{(1+h)^3 - 1}{h}$.

Solution: Expand the numerator to get $\lim_{h\to0} \frac{1+3h+3h^2+h^3-1}{h} = \lim_{h\to0} \frac{3h+3h^2+h^3}{h} = \lim_{h\to0} \frac{h(3+3h+h^2)}{h} = \lim_{h\to0}(3 + 3h + h^2$, and now the zero over zero problem has disappeared. So the limit is 3.

For problems (g) through (m), let

$$f(x) = \begin{cases} -2 & \text{if } x < 0 \\ 2x - 2 & \text{if } 0 \le x < 2 \\ 3 & \text{if } x = 2 \\ 7 - 2x & \text{if } x > 2 \end{cases}$$

(g) $\lim_{x \to 0^-} f(x)$

Solution: -2

(h) $\lim_{x \to 0^+} f(x)$

Solution: -2

(i) $\lim_{x \to 0} f(x)$

Solution: -2

(j) $f(0)$

Solution: -2

(k) $\lim_{x \to 2^-} f(x)$

Solution: 2

(l) $\lim_{x \to 2^+} f(x)$

Solution: 3

(m) $\lim_{x \to 2} f(x)$

Solution: DNE

3. (12 points) The demand curve for a certain item is given by $p = -x^2 - 2x + 100$ where x represents the quantity demanded in units of a thousand and p represents the price in dollars. The supply curve is given by $p = 8x + 25$. Find the equilibrium quantity and equilibrium price.

   Solution: At the equilibrium point, the two curves intersect, so write $-x^2 - 2x + 100 = 8x + 25$, which is equivalent to $x^2 + 10x - 75 = 0$. Factor the left side to get $(x-5)(x+15) = 0$, and discard the negative root. So $x = 5$ thousand and $p = 8 \cdot 5 + 25 = 65$.

4. (15 points) The function $f(x) = \frac{1}{1+\frac{1}{x}}$ is continuous for all $x > 0$. Let $a = 1$.

   (a) Pick a number $b > 1$ (any choice is right), and then find a number M between $f(a)$ and $f(b)$.
   Solution:

   (b) Show that the conclusion to the Intermediate Value Theorem is satisfied by finding a number c in (a, b) such that $f(c) = M$.
   Solution:

5. (8 points) Find all the x-intercepts of the function

$$g(x) = 3(2x-5)^3(2x+1)^2 - 6(2x-5)^2(2x+1)^3.$$

   Solution: Factor the common stuff out to get $(2x+1)^2(2x-5)^2[3(2x-5) - 6(2x+1)]$. Setting each of the three factors to zero yields $x = -1/2, x = 5/2$, and $x = -7/2$.

6. (15 points)

   (a) Find all solutions of the equation $|| x - 3 | -5 | = 1$.

   Solution: Note that either $| x - 3 | -5 = 1$ or $| x - 3 | -5 = -1$. In the first case, $| x - 3 | = 6$ in which case $x - 3 = 6$ or $x - 3 = -6$. Thus both $x = 9$ and $x = -3$ are solutions. In the second case $| x - 3 | = 4$ in which case $x - 3 = 4$ or $x - 3 = -4$. So we get $x = -1$ and $x = 7$.

   (b) Find the (implied) domain of

$$f(x) = \sqrt{|| x - 3 | -5 | -1},$$

   and write your answer in interval notation.

   Solution: Note that the domain must include those values of x for which the value inside the radical is at least zero. Use the Test Interval Technique with the numbers $\{-3, -1, 7, 9\}$ as branch points. We can use test points $x = -2$ and $x = 7$ to see that $|| x-3 | -5 | -1$ is negative in the intervals $(-3, -1)$ and $(7, 9)$. So the domain D is $(-\infty, -3] \cup [-1, 7] \cup [9, \infty)$

7. (20 points) Let $f(x) = x^2 - x$. Note that $f(3) = 6$

   (a) Find the slope of the line joining the points $(3, 6)$ and $(3 + h, f(3 + h))$, where $h \neq 0$. Note that $(3 + h, f(3 + h))$ is a point on the graph of f.

   Solution: .

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   Solution:

$$\begin{aligned} k'(x) &= \lim_{h\to 0} \frac{k(x+h) - k(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2 - h}{h} \\ &= \lim_{h\to 0} \frac{h(2x + h - 1)}{h} = 2x - 1. \end{aligned}$$

   (c) Replace the x with 3 in your answer to (b) to find $f'(3)$.

   Solution: $f'(3) = 5$

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(3, 6)$.

   Solution: The line is $y - 6 = 5(x - 3)$, or $y = 5x - 9$.

8. (12 points) Given two functions,

$$g(x) = \begin{cases} 2x - 1 & \text{if } 1 < x < 4 \\ 4 - x & \text{otherwise} \end{cases} \quad \text{and} \quad f(x) = \begin{cases} x^2 + 3 & \text{if } x \geq 1 \\ x^2 - 4 & \text{if } x < 1 \end{cases}$$

Complete the following table.

x	$g(x)$	$f(x)$	$f \circ g(x)$	$g \circ f(x)$
-4	8	12	67	-8
-1				
0				
1				
2				
3				
3.5				
4				

Solution:

x	$g(x)$	$f(x)$	$f \circ g(x)$	$g \circ f(x)$
-4	8	12	67	-8
-1	5	-3	28	7
0	4	-4	19	8
1	3	4	12	0
2	3	7	12	-3
3	5	12	28	-8
3.5	6	61/4	39	-45/4
4	0	19	-4	-15