

October 15, 2008

Name _____

The problems count as marked. The total number of points available is 133. Throughout this test, **show your work.**

- (6 points) Find an equation (in slope-intercept form) for a line parallel to the line $3x - 6y = 7$ and which goes through the point $(-3, 5)$.

- (40 points) Evaluate each of the limits (and function values) indicated below.

(a) $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^2 - 4x + 3}$

(b) $\lim_{x \rightarrow 3} \frac{x - 3}{\frac{1}{x} - \frac{1}{3}}$

(c) $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^2 - 3}}{11 - 5x}$

(d) $\lim_{x \rightarrow \infty} \frac{6x^5 - 3x^3}{11 - 12x^4}$

(e) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1}$

(f) $\lim_{h \rightarrow 0} \frac{(2 + h)^3 - 8}{h}$.

The following eight problems are worth 2 points each. For problems (g) through (n), let

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x - 1 & \text{if } 0 \leq x < 2 \\ -1 & \text{if } x = 2 \\ 1 & \text{if } x > 2 \end{cases}$$

Find the value, if it exists, of each item below. Use DNE when the limit does not exist.

(g) $\lim_{x \rightarrow 0^-} f(x)$

(h) $\lim_{x \rightarrow 0^+} f(x)$

(i) $\lim_{x \rightarrow 0} f(x)$

(j) $f(0)$

(k) $\lim_{x \rightarrow 2^-} f(x)$

(l) $\lim_{x \rightarrow 2^+} f(x)$

(m) $\lim_{x \rightarrow 2} f(x)$

(n) $f(2)$

3. (10 points) Find all the x -intercepts of the function

$$g(x) = 3(2x + 7)^2(x - 1)^2 - (2x + 7)(x - 1)^3.$$

4. (15 points)

- (a) Find all solutions of the inequality $|2x - 7| \leq 5$ and write your solution in interval notation.

- (b) Find the (implied) domain of

$$f(x) = \sqrt{|2x - 7| - 3},$$

and write your answer in interval notation.

5. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

(a) Find the slope of the line joining the points $(0, 1)$ and $(0 + h, f(0 + h)) = (h, f(h))$, where $h \neq 0$.

(b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

(c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

(d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(0, 1)$.

6. (18 points) If a ball is thrown vertically upward from the roof of 128 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 128 + 64t - 16t^2$.

(a) What is the height the ball at time $t = 1$?

(b) What is the velocity of the ball at the time it reaches its maximum height?

(c) What is the maximum height the ball reaches?

(d) After how many seconds is the ball exactly 160 feet above the ground?

(e) How fast is the ball going the first time it reaches the height 160?

(f) How fast is the ball going the second time it reaches the height 160?

7. (24 points) Compute the following derivatives.

(a) Let $f(x) = \frac{x^2 - 2x}{3x - x^2}$. Find $\frac{d}{dx} f(x)$.

(b) Let $g(x) = \sqrt{x^3 + 2x + 4}$. What is $g'(x)$?

(c) Find $\frac{d}{dx}((3x + 1)^2 \cdot (4x^2 - 1))$

(d) Let $f(x) = (2x^2 + 1)^4$. Find $f'(x)$.