February 12, 2009 **Name** _____

The problems count as marked. The total number of points available is 180. Throughout this test, **show your work.**

1. (8 points) Find the exact value of the expression $|10-3\sqrt{5}|-|2\sqrt{5}-4|-|\sqrt{5}-6|$. Express your answer in a very simple form.

2. (8 points) Find an equation for a line perpendicular to the line $3x - 2y = 7$ and which goes through the point $(-3, 5)$.

3. (52 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to 1} \dfrac{x^2 + 2x - 3}{x^2 - 6x + 5}$

   (b) $\lim_{x\to 3} \dfrac{\frac{2}{x} - \frac{2}{3}}{x - 3}$

   (c) $\lim_{x\to -\infty} \dfrac{|\ 18x - 3\ |}{6x - 11}$

   (d) $\lim_{x\to \infty} \dfrac{6x^4 - 3}{(11 - 3x^2)^2}$

   (e) $\lim_{x\to -1} \dfrac{x^3 + 1}{x^2 - 1}$

   (f) $\lim_{h\to 0} \dfrac{(1 + h)^3 - 1}{h}$.

For problems (g) through (m), let

$$f(x) = \begin{cases} -2 & \text{if } x < 0 \\ 2x^2 - 2 & \text{if } 0 \le x < 2 \\ 3 & \text{if } x = 2 \\ 10 - 2x & \text{if } x > 2 \end{cases}$$

(g) $\lim_{x \to 2^-} f(x)$

(h) $\lim_{x \to 2^+} f(x)$

(i) $\lim_{x \to 2} f(x)$

(j) $\lim_{x \to 0^-} f(x)$

(k) $\lim_{x \to 0^+} f(x)$

(l) $\lim_{x \to 0} f(x)$

(m) $f(0)$

4. (12 points) The demand curve for a certain item is given by $p = -x^2 - 8x + 100$ where x represents the quantity demanded in units of a thousand and p represents the price in dollars. The supply curve is given by $p = 4x + 20$. Find the equilibrium quantity and equilibrium price.

5. (10 points) Find all the x-intercepts of the function

$$g(x) = (2x^2 - 1)^2(3x + 1) - (2x^2 - 1)(3x + 1)^2.$$

6. (30 points) Let $g(x) = \sqrt{\frac{2x-7)(3x+4)}{x^2-6x+5}}$. The sequence of steps below will enable you to find the (implied) domain of g. Let $r(x) = (g(x))^2 = \frac{(2x-7)(3x+4)}{x^2-6x+5}$.

   (a) Find the zeros of r.

   (b) Find the value(s) of x for which r is undefined.

   (c) Write as a union of intervals the set of real numbers that result by removing the values of x found in the first two parts.

   (d) For each of the intervals in part 3, select a point in the interval, and compute the sign (plus or minus) of r at that test point.

   (e) Express the domain of $g(x)$ as a union of intervals. Be sure to include or exclude the endpoints as appropriate.

7. (20 points) Let $f(x) = x^2 - 2x$. Note that $f(2) = 0$

   (a) Find the slope of the line joining the points $(2, 0)$ and $(2 + h, f(2 + h))$, where $h \neq 0$. Note that $(2 + h, f(2 + h))$ is a point on the graph of f. Then find the limit of the expression as h approaches 0 to compute $f'(2)$.

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   (c) Replace the x with 2 to find $f'(2)$.

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(2, 0)$.

8. (40 points) Below is a table of some of the values of two functions f and g and information about their some of their left-hand and right-hand limits. All the questions below refer to values of a in the set $\{-2, -1, 0, 1, 2, 3\}$.

a	$f(a)$	$g(a)$	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a^-} g(x)$	$\lim_{x\to a^+} g(x)$
-2	1	2	1	1	2	DNE
-1	0	1	2	2	1	1
0	2	-1	DNE	0	-1	-1
1	-1	0	-1	-1	2	-2
2	-2	-1	-2	-1	-1	-1
3	1	1	1	1	1	-1

(a) For which values of a does $\lim_{x\to a} f(x)$ exist?

(b) For which values of a does $\lim_{x\to a} g(x)$ exist?

(c) For which values of a is $f(x)$ continuous?

(d) For which values of a is $g(x)$ continuous?

(e) Find each of the following, if they exist.

i. $\lim_{x\to -1}[f(x)\cdot g(x)]$.

ii. $f \circ g \circ f(1)$

iii. $g \circ g \circ g(1)$

iv. $g(\lim_{x\to -1} f(x))$.

(f) Find a value of a satisfying each of the equations. If more than one value exists, find them all.

i. $f \circ g(a) = 0$.

ii. $g \circ f(a) = 0$.

iii. $(f(a))^2 + (g(a))^2 = 5$.

iv. $(\lim_{x\to a} f(x))^2 + (\lim_{x\to a} g(x))^2 = 5$.