October 15, 2009 **Name** ______________________

The problems count as marked. The total number of points available is 163. Throughout this test, **show your work.**

1. (8 points) Find the exact value of the expression $|\pi - 7| + |2\pi - 10| + |3\pi - 8|$. Express your answer in a very simple form.

   Solution: Solve each absolute value separately to get $7 - \pi$, $10 - 2\pi$ and $3\pi - 8$. Therefore, the sum is $7 - \pi + 10 - 2\pi + 3\pi - 8 = 17 - 8 = 9$.

2. (8 points) Find an equation for a line perpendicular to the line $5x - 2y = 7$ and which goes through the point $(-3, 9)$. Express your answer in slope-intercept form.

   Solution: The given line has slope $5/2$ so the one perpendicular has slope $-2/5$. Hence $y - 9 = (-2/5)(x + 3)$. Thus $y = -2x/5 + 39/5$.

3. (30 points) Evaluate each of the limits indicated below.

   (a) $\lim\limits_{x\to\infty} \dfrac{3x^4 - 6}{(11 - 3x^2)^2}$

   Solution: The degrees of the numerator and denominator are both 4 so the limit is $3/9 = 1/3$.

   (b) $\lim\limits_{x\to 1} \dfrac{x^3 - 1}{x^2 - 1}$

   Solution: Factor both numerator and denominator to get $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = 3/2$

   (c) $\lim\limits_{h\to 0} \dfrac{(2 + h)^3 - 8}{h}$.

   Solution: Expand the numerator to get

   $$\lim_{h\to 0} \frac{8 + 12h + 6h^2 + h^3 - 8}{h} = \lim_{h\to 0} \frac{12h + 6h^2 + h^3}{h} = \lim_{h\to 0} \frac{h(12 + 6h + h^2)}{h}$$

   $= \lim\limits_{h\to 0}(12+6h+h^2)$, and now the zero over zero problem has disappeared. So the limit is 12.

   (d) $\lim\limits_{x\to 1} \dfrac{x^2 - 4x + 3}{x^2 + x - 2}$

   Solution: Factor and eliminate the $x - 1$ from numerator and denominator to get

   $$\lim_{x\to 1} \frac{x - 3}{x + 2} = -2/3$$

(e) $\lim_{x\to 3} \dfrac{\frac{4}{x} - \frac{4}{3}}{x-3}$

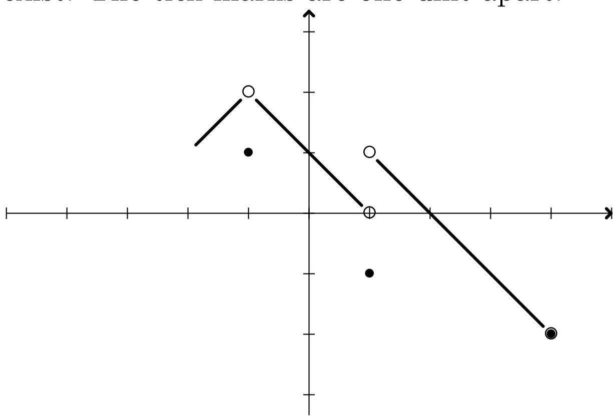
Solution: The limit of both the numerator and the denominator is 0, so we must do the fractional arithmetic. The limit becomes

$$\lim_{x\to 3} \frac{\frac{4(3-x)}{3x}}{(x-3)} = \lim_{x\to 3} \frac{-\frac{4(x-3)}{3x}}{x-3} = \lim_{x\to 3} \frac{-\frac{4}{3x}}{1} = -4/9.$$

(f) $\lim_{x\to -\infty} \dfrac{\sqrt{36x^2-3}}{9x-11}$

Solution: Divide both numerator and denominator by x to get $\lim_{x\to-\infty} \frac{\sqrt{36-3/x^2}}{9-11/x} = 6/9 = 2/3$ because the degree of the denominator is essentially the same as that of the numerator.

4. (18 points) Consider the function F whose graph is given below. Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist. The tick marks are one unit apart.



(a) $\lim_{x\to -1^-} F(x) =$

Solution: 2

(b) $\lim_{x\to -1^+} F(x) =$

Solution: 2

(c) $\lim_{x\to -1} F(x) =$

Solution: 2

(d) $F(-1) =$

Solution: 1

(e) $\lim_{x\to 1^-} F(x) =$

Solution: 0

(f) $\lim_{x\to 1^+} F(x) =$

Solution: 1

(g) $\lim_{x\to 1} F(x) =$

Solution: dne

(h) $\lim_{x\to 3} F(x) =$

Solution: -1

(i) $F(3) =$

Solution: -1

5. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{x+1}}{(x-1)(x-3)}.$$

Express your answer as a union of intervals. That is, use interval notation.

Solution: The numerator is defined for $x + 1 \geq 0$, that is $[-1, \infty)$. The denominator is zero at $x = 1$ and at $x = 3$, so these two numbers must be removed from $[-1, \infty)$. Thus, the domain is $[-1, 1) \cup (1, 3) \cup (3, \infty)$.

6. (12 points) The demand curve for a certain item is given by $p = x^2 - 15x + 98$ where x represents the quantity demanded in units of a thousand and p represents the price in dollars. The supply curve is given by $p = 4x + 50$. Find the equilibrium quantity and equilibrium price.

Solution: Solve the two equations simultaneously by setting $x^2 - 15x + 98$ equal to $4x + 50$. This results in the quadratic $x^2 - 19x + 48 = 0$, which we can solve by factoring, $x = 3$, and $x = 16$. So $p = 4 \cdot 3 + 50 = 62$. Or $p = 4 \cdot 16 + 50 = 114$.

7. (10 points) Find all the x-intercepts of the function

$$g(x) = (2x^2 - 1)^2(3x + 1) - (2x^2 - 1)(3x + 1).$$

Solution: Factor out the common terms to get $g(x) = (2x^2 - 1)(3x + 1)[(2x^2 - 1) - 1] = (2x^2 - 1)(3x + 1)(2x^2 - 2)$. Setting each factor equal to zero, we find the zeros are $x = -\sqrt{2}/2, x = \sqrt{2}/2$, $x = -1/3, x = 1$ and $x = -1$.

8. (30 points) Let $g(x) = \sqrt{\frac{(2x-15)(3x+17)}{x^2+x-6}}$. The sequence of steps below will enable you to find the (implied) domain of g. Let $r(x) = (g(x))^2 = \frac{(2x-15)(3x+17)}{x^2+x-6}$.

   (a) Find the zeros of r. That is, find all x for which $r(x) = 0$.

   Solution: Solve $2x - 15 = 0$ to get $x = 15/2$ and solve $3x + 17 = 0$ to get $x = -17/3$.

   (b) Find the value(s) of x for which r is undefined.

   Solution: Solve $x^2 + x - 6 = 0$ to get $x - 2 = 0$ or $x = 2$ and $x + 3 = 0$ to get $x = -3$.

   (c) Write as a union of intervals the set of real numbers that result by removing the values of x found in the first two parts.

   Solution: It is $(-\infty, -17/3) \cup (-17/3, -3) \cup (-3, 2) \cup (2, 15/2) \cup (15/2, \infty)$.

   (d) For each of the intervals in part (c), select a test point in the interval, and compute the sign (plus or minus) of r at that test point.

   Solution:

   (e) Express the domain of $g(x)$ as a union of intervals. Be sure to include or exclude the endpoints as appropriate.

   Solution: Since $r(x)$ is positive on the first third and fifth of the intervals, our answer is at least $(-\infty, -17/3) \cup (-3, 2) \cup (15/2, \infty)$. But the endpoints $15/2$ and $-17/3$ must be included also. Thus we have Domain of g:$(-\infty, -17/3] \cup (-3, 2) \cup [15/2, \infty)$.

9. (25 points) Let $f(x) = \sqrt{3x-2}$. Notice that $f(6) = \sqrt{18-2} = 4$.

   (a) Find the slope of the line joining the points $(6, 4)$ and $(6+h, f(6+h))$, where $h \neq 0$. Note that $(6+h, f(6+h))$ is a point on the graph of f.
   Solution: $\frac{\sqrt{3(6+h)-2}-4}{6+h-6} = \frac{\sqrt{3(6+h)-2}-4}{h}$.

   (b) Compute $f(a+h)$, $f(a)$, and finally $\frac{f(a+h)-f(a)}{h}$.
   Solution:

   (c) Finally compute the limit as h approaches 0 to find $f'(a)$.
   Solution: Since we get zero over zero, we recall that, in this case, we should rationalize the numerator.

$$\begin{aligned}
\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0} \frac{\sqrt{3(a+h)-2}-\sqrt{3a-2}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3(a+h)-2}-\sqrt{3a-2}}{h} \cdot \frac{\sqrt{3(a+h)-2}+\sqrt{3a-2}}{\sqrt{3(a+h)-2}+\sqrt{3a-2}} \\
&= \lim_{h\to 0} \frac{3(a+h)-2-(3a-2)}{h(\sqrt{3(a+h)-2}+\sqrt{3a-2})} \\
&= \lim_{h\to 0} \frac{3h}{h(\sqrt{3(a+h)-2}+\sqrt{3a-2})} \\
&= \lim_{h\to 0} \frac{3}{(\sqrt{3(a+h)-2}+\sqrt{3a-2})} \\
&= \frac{3}{2(\sqrt{3a-2})}
\end{aligned}$$

   (d) Replace the a with 6 to find $f'(6)$.
   Solution: $f'(6) = 3 \cdot 16^{-1/2}/2 = 3/8$

   (e) Use the information given and that found in (d) to find an equation for the line tangent to the graph of f at the point $(6, 4)$.
   Solution: The line is $y - 4 = 3(x-6)/8$, or $y = 3x/8 + 7/4$.

10. (10 points) Write in interval form the set of all real numbers x for which

$$f(x) = \frac{|\, x-1 \,|}{x-1} + \frac{|\, x+3 \,|}{x+3}$$

is continuous.

Solution: We must eliminate precisely the values $x = 1$ and $x = -3$, so the set is $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.