February 17, 2010 **Name**

The problems count as marked. The total number of points available is 156. Throughout this test, **show your work.**

1. (10 points) Find an equation for a line that is perpendicular to the line $2x - 3y = 7$ and which passes through the point $(4, 2)$. Write your answer in slope-intercept form.

2. (20 points) Let $f(x) = (2x-3)^4(5x-1)^2 + 17x^2$, and let $g(x) = (x-4)^3(8x^3) - 17x^2$.

   (a) What is the degree of the polynomial f?

   (b) What is the degree of the polynomial g?

   (c) Estimate within one unit the value of $f(1000)/g(1000)$.

   (d) Compute $\lim_{x \to \infty} \frac{f(x)}{g(x)}$.

3. (15 points) Find the (implied) domain of each of the functions given below. Express your answers in interval notation.

(a) $f(x) = \frac{1}{x^2 - 9}$

(b) $g(x) = \sqrt{x - 4}$

(c) $h(x) = \sqrt{x(x - 1)(x + 3)}$

4. (55 points) Evaluate each of the limits indicated below.

(a) $\lim_{x \to 0} \frac{x^4 - x^2}{x^2}$

(b) $\lim_{x \to 3} \frac{\frac{1}{3x} - \frac{1}{9}}{x - 3}$

(c) $\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5}$

(d) $\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}$

(e) $\lim_{x \to \infty} \frac{\sqrt{16x^2 - 3}}{11 - 5x}$

For problems (f) through (k), let

$$f(x) = \begin{cases} 7 - x & \text{if } x > 2 \\ 10 & \text{if } x = 2 \\ 2x + 1 & \text{if } 0 \le x < 2 \\ -1 & \text{if } x < 0 \end{cases}$$

(f) $\lim\limits_{x \to 0^-} f(x)$

(g) $\lim\limits_{x \to 0^+} f(x)$

(h) $\lim\limits_{x \to 0} f(x)$

(i) $\lim\limits_{x \to 2^-} f(x)$

(j) $\lim\limits_{x \to 2^+} f(x)$

(k) $\lim\limits_{x \to 2} f(x)$

5. (21 points) Consider the function whose properties are displayed.

a	-1	0	1	2	3	4
$\lim\limits_{x \to a^-} f(x)$	DNE	2	2	4	2	3
$\lim\limits_{x \to a^+} f(x)$	1	2	2	3	2	DNE
$f(a)$	1	2	-1	1	2	3
$\lim\limits_{x \to a^-} g(x)$	4	3	3	3	-1	0
$\lim\limits_{x \to a^+} g(x)$	1	-2	0	3	-1	DNE
$g(a)$	-1	-1	3	-3	DNE	0

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a) $\lim\limits_{x \to 2^+} [f(x) - g(x)]$

(b) $\lim\limits_{x \to 2^-} [f(x) - g(x)]$

(c) $\lim\limits_{x \to 2} [f(x) - g(x)]$

(d) $(f + g)(4)$

(e) $f \circ g \circ f(-1)$

(f) Find all points (in the table) at which f is continuous.

(g) Find all points (in the table) at which g is continuous.

6. (10 points) Find all the x-intercepts of the function

$$g(x) = (2x^2 - 4)^2(3x + 2) + (2x^2 - 4)^3(3x + 2).$$

7. (25 points) Let $f(x) = \sqrt{3x - 2}$. Notice that $f(6) = \sqrt{18 - 2} = 4$.

   (a) Find the slope of the line joining the points $(6, 4)$ and $(6 + h, f(6 + h))$, where $h \neq 0$. Note that $(6 + h, f(6 + h))$ is a point on the graph of f.

   (b) Compute $f(a + h), f(a)$, and finally $\frac{f(a+h)-f(a)}{h}$.

   (c) Finally compute the limit as h approaches 0 to find $f'(a)$.

   (d) Replace the a with 6 to find $f'(6)$.

   (e) Use the information you found in part (d) to find an equation for the line tangent to f at the point $(6, f(6))$.