

October 4, 2011

Name _____

The problems count as marked. The total number of points available is 151.

Throughout this test, **show your work.**

1. (10 points) What is the exact value of $|8 - 2\pi| + |2\pi - 7|$. A solution that fails to show your understanding of the definition of absolute value is worth at most 1 point.

2. (12 points)

(a) For what value of k does the line $2y + kx = 6$ go through the point $(1, 4)$?

(b) Find the slope-intercept form of the line perpendicular to the line in (a) that includes the point $(1, 4)$.

3. (15 points) Find the domain and range of each of the three functions below. Express your answers using interval notation. Use the letters D and R for domain and range, respectively.

(a) $h(x) = \frac{x^2-1}{x-1}$.

(b) $g(x) = \frac{\sqrt{2-x}}{x+4}$

(c) $f(x) = \begin{cases} x - 3 & \text{if } x < 2 \\ x + 2 & \text{if } x > 2 \end{cases}$

4. (52 points) Evaluate each of the limits indicated below.

$$(a) \lim_{x \rightarrow \infty} \frac{3x^4 - 6}{(11 - 3x^2)^3}$$

$$(b) \lim_{x \rightarrow 1} \frac{x^4 - 1}{x^2 - 1}$$

$$(c) \lim_{x \rightarrow 1} \frac{(x - 2)^3 + 1}{x - 1}$$

$$(d) \lim_{x \rightarrow -2} \frac{x^2 + 2x}{x^2 + 5x + 6}$$

$$(e) \lim_{x \rightarrow 2} \frac{\frac{1}{4x} - \frac{1}{8}}{\frac{1}{2x} - \frac{1}{4}}$$

$$(f) \lim_{x \rightarrow 8} \frac{\sqrt{x+1} - 3}{x - 8}$$

For problems (g) through (m), let

$$f(x) = \begin{cases} -2 & \text{if } x < 0 \\ 2x^2 - 2 & \text{if } 0 \leq x < 2 \\ 3 & \text{if } x = 2 \\ 10 - 3x & \text{if } x > 2 \end{cases}$$

(g) $\lim_{x \rightarrow 2^-} f(x)$

(h) $\lim_{x \rightarrow 2^+} f(x)$

(i) $\lim_{x \rightarrow 2} f(x)$

(j) $\lim_{x \rightarrow -1} f(x)$

(k) $\lim_{x \rightarrow 3} f(x)$

(l) $\lim_{x \rightarrow 0} f(x)$

(m) Tell whether the function is continuous at each of the points:

i. $x = 0$

ii. $x = 1$

iii. $x = 2$

iv. $x = 3$

5. (12 points) Let $H(x) = (x^2 - 1)(x + 2)^3$. Using the product rule,

$$H'(x) = (2x) \cdot (x + 2)^3 + 3(x^2 - 1) \cdot (x + 2)^2.$$

Find the three zeros of $H'(x)$.

6. (12 points) Let $f(x) = 2x - \frac{1}{x}$ and let $g(x) = x^2 - 2$. Compute the composite functions listed below.

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

7. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

(a) Find the slope of the line joining the points $(0, 1)$ and $(0+h, f(0+h)) = (h, f(h))$, where $h \neq 0$. Then find the limit as h approaches 0 to get $f'(0)$.

(b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

(c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

(d) Use the information given and that found in (c) to find an equation in slope-intercept form for the line tangent to the graph of f at the point $(0, 1)$.

8. (18 points) If a ball is thrown vertically upward from the roof of 128 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 128 + 64t - 16t^2$.
- (a) What is the height the ball at time $t = 1$?

 - (b) What is the velocity of the ball at the time it reaches its maximum height?

 - (c) What is the maximum height the ball reaches?

 - (d) After how many seconds is the ball exactly 160 feet above the ground?

 - (e) How fast is the ball going the first time it reaches the height 160?

 - (f) How fast is the ball going the second time it reaches the height 160?